NB1140 - Physics 1A
Homework Set 3
Due: 08:45hr on *Tuesday 1st December 2015* (Before the Analysis 2 lecture)

This homework set covers the following topics:

- Center of mass
- Momentum
- Collisions (conservation of momentum, *sometimes* conserved kinetic energy, *always* conserved total energy).

Total = 50 points

A. Conceptual questions. (*Total = 10 points*)
Clearly and concisely explain the reasons that led to your final answers (even for those whose answer is "yes" or "no"). A correct solution can be as long as one or two sentences that hit the right points. Some questions require short calculations. Be sure to show all the steps to receive a full credit.

Question 1. (*1 points*)
Why is the concept of the center of mass useful?

Question 2. (*1 points*)
Can the center of mass lie outside any object that makes up the system? If yes, give an example. If no, explain why not.

Question 3. (*1 points*)
Consider a rod of uniform mass density. It expands so that it doubles in length but still remains a 1-dimensional rod. Where does the expanded rod's center of mass in comparison to its position before the rod expanded?

Question 4. (*3 points*)
Consider two objects that have the same mass. They are moving towards each other with the same constant speed. When they collide, they stick to each other. What is each block's kinetic energy after they collide? Has any kinetic energy been lost in this collision? If not, explain why not. If any kinetic energy is lost, explain where the energy went.

Question 5. (*4 points*)
If you throw a ball against a hard wall of your room, the ball bounces back. Suppose you throw a tennis ball straight at a wall with velocity V. We showed in class that this ball should bounce back directly with velocity $-V$. In other words, just the direction of the velocity is reversed but the magnitude of the two velocities (i.e., the speeds before and after the bounce) are the same. Paradoxically, this seems to violate the conservation of momentum. Before the collision, the total momentum of the system (wall and the ball) seems to be mV. After the collision, the total momentum of the system seems to be -mV. Explain what is wrong with this reasoning and why the total momentum is conserved. Where did the "hidden" momentum go?

B. Problems that require longer calculations. (*Total = 40 points*)
In order to receive full credit, be sure to show all the steps and calculations that led you to the final answer. You will receive zero points if you just write the final answer without showing how you got your answer. Almost all the questions below require very few lines of calculations. Long and convoluted reasoning with many unnecessary calculations will get lower number of points than a concise and logical (step-by-step) presentation of calculations that are required to get to the final answer.

Problem 1. (Total = 10 points) Slowing down an object by shooting balls at it
Intuition tells you that you can shoot at an object to either push it forward. Perhaps less familiar to you is the idea that you can slow down an object by shooting at the object to slow it down. Let's quantify this intuition with some calculations.

a.) (*4 points*) A car of mass *M* moves to the right with a constant velocity $\vec{V}$ and collides with a tennis ball of mass *m* that is moving to the left with a constant velocity $\vec{U}$. After the two bodies collide, the ball elastically bounces off the car and moves to the right (i.e., total kinetic energy is conserved through the whole process). Show that the car must move slower after the collision than it did before the collision.
[*Hint: If you find yourself writing down and trying to solve many equations, stop and think. Consider conservation of momentum and the fact that you need to consider adding two vectors.*]

b.) (*2 points*) Can the car move faster after the collision if the collision were inelastic? To be concrete, assume that the tennis ball sticks to the car's windshield upon colliding (or, if a sticky tennis ball sounds weird, you can assume that a snowball of mass *m* sticks to the windshield after colliding). [*Hint: What happens to the total momentum of the system in inelastic collisions?*]

c.) (*4 points*) In the above case of the snowball sticking to the car after the collision, what is the kinetic energy of the system (car + snowball) before the collision? What is it after the collision?

Problem 2. (Total = 20 points) A chain falling on top of a weighing scale.
Consider a chain of length *L* and mass *M.* To be concrete, let's say this is a bicycle lock, the kind that you wrap around your bicycle and a pole, that is in the unlocked (open) form. Suppose the chain has a uniform mass density (i.e., the total mass is uniformly distributed along the length *L*). You hold one end of the chain with your hand above a weighing scale so that the chain is straight and the other end of the chain is just above the weighing scale without touching it.

a.) (*2 points*) Where is the center of mass of the chain? Give the distance above the weighing scale.

Initially the chain is at rest. It's suspended in the air by your fingertips. You release the chain so that it freely falls on top of the weighing scale. In this problem, you will calculate what weight the weighing scale will read as a function of the length of chain that has landed on the scale.

b.) (*3 points*) At time *t*, let *y* be the length of the chain that has already landed on the weighing scale $(0 \leq y \leq L)$. Where is the center of mass now? You can assume that the part of the chain that has landed on the weighing scale lies flat on the scale. You can imagine the chain to form an "L" shape. Why isn't the answer just $(L - y)/2$?
[*Hint: When you derive your final answer (formula), one way to check if your formula makes sense is by putting in "extreme values" into your formula. Extreme values here would be y=0 and y=L. You already know where the center of mass would be for these two values of y. So your formula should give you the same answer.*]

c.) (*4 points total*) What is the total momentum of the system (composed of the part on the scale and the part in the air) when *y* is the length of chain that's resting on the scale? Calculate this in two ways.

1. (*2 points*) Break the chain into two "objects". One object sits on scale and the other object is freely falling (not yet touching the table). Calculate the momentum of each then sum them. This is the total momentum of the system. Your answer should contain derivative(s) that you cannot evaluate yet.
2. (*2 points*) Use the definition of center of mass. Remember that we can consider the whole system to be a single point particle whose mass is the total mass of the chain *M*. Again, leave your answer in terms of derivative(s) that you cannot evaluate yet.

d.) (*2 points*) Recall that the true definition of "force" is the rate of change of the momentum *P* with respect to time. Considering the chain as a point (i.e., the center of mass), write down an equation that relates *dP/dt* to all the forces acting on the chain. Define what each force is. [*Hint: You'll just write dP/dt on one side of the equation and on the other side, you'll write down all the forces acting on the chain. Be sure to specify what each force is and in which direction the force vector is pointing*].

e.) (*1 points*) Let *Y* = *L-y*. By evaluating *dP/dt*, derive an expression for the force that the scale exerts on the chain. Leave your answer in terms of *dY/dt.* Be careful with the signs in front of all the terms (*Hint: Think about whether "up" is + or - in your calculations so far -- always be careful with the sign convention you're using in your problem. Pick which direction will be + or - and stick with it throughout the problem.]*

f.) (*2 points)* One of the joys of calculations in physics is that many of the terms that you cannot explicitly calculate but still appear in your equations disappear at the end of the day. Let's see this in action. At the moment that the top end of the chain hits the ground, show that the force that the scale exerts on the chain at this moment is:

$$Mg + \frac{M}{L}\left(\frac{dY}{dt}\right)^2$$

g.) (*2 points*) Every segment of the chain is falling freely under the influence of gravity until it fits the scale. We can consider the chain to be made of infinitesimal pieces that are joined together (e.g., tiny beads of mass *dm* that are joined together in a line). Seen in this way, each piece of mass *dm* falls freely before it hits the ground. Considering the motion of the piece *dm* that is at the top of the chain just before it hits the scale, show that

$$\frac{dY}{dt} = \sqrt{2gL}$$

Be sure to justify your answer. "Working backwards" won't get you any points.

h.) (*2 points*) Let's put it altogether. Show that the weighing scale should read "*3Mg*" at the moment that the top of the chain hits it.
(*This is incredible! Three times the total weight of the chain is felt by the scale at the moment the top part of the chain lands on it! This is why it is very dangerous to be hit by a falling object, even things that seem "light" when you hold them in your hand.)*

i.) (*2 points*) Why is the force exerted by the scale so large? Give a physical / intuitive reasoning (*Hint: what is the scale doing to the chain when different parts of the chain are hitting the scale?)*

j.) (*Optional: 4 bonus points*) Part (g) suggests an alternate method for solving this problem. Without using the concept of center of mass at all, derive a more general result, namely an equation that describes the scale reading (i.e., the weight felt by the scale) as a function of *y*. Of course, to check that you derived the correct formula, you should check that your formula gives you the *3Mg* when *y* = *L*.

Problem 3. (Total = 10 points) Shooting from a spaceship

Consider a spaceship of mass *M*. The spaceship is carrying a ball of mass *m* from its side. *M* does not include *m*, so the initial total mass of the system (ship + ball) is *M+m*. The spaceship and all its contents (including the ball) move with a constant velocity $\vec{V}$ out in deep space. There is no gravity or any other force acting on the spaceship (i.e., the spaceship's engines are turned off, there's no friction or gravity in space). To be concrete, let's say that $\vec{V}$ is a vector that is pointing in the "North" direction (in space, there's no North, South, East, or West, but let's ignore this fact).

a.) (*3 points*) Assume that you're an observer floating in space. The spaceship is moving at velocity $\vec{V}$ in the North direction relative to you. Describe the motion of the center of mass (it's position, velocity, and acceleration as a function of time *t* as you would observe it. *Write an equation for each one as a function of time.*

b.) (*4 points*) Now you're an observer in a separate spaceship that is travelling at the same constant velocity $\vec{V}$ as the spaceship. You're watching the spaceship. You see that the spaceship shoots out the ball of mass *m* directly in the West direction with a constant velocity $\vec{U}$. Note that $\vec{U}$ is the velocity of the ball that you measure while you're travelling in your spaceship. What is the new velocity and position of the spaceship.

c.) (*3 points*) How would an observer who is floating still, instead of moving at velocity $\vec{V}$, describe the motion of the spaceship after the ball is launched from the ship? Describe its position and velocity as a function of time as seen by such an observer.