NB1140 - Physics 1A
Homework Set 6
Due: 08:45hr on *Tuesday 12th January 2016* (Before the Analysis 2 lecture)

Problem 1. Motion of a spinning top with two forces acting on it

[*10 points*]

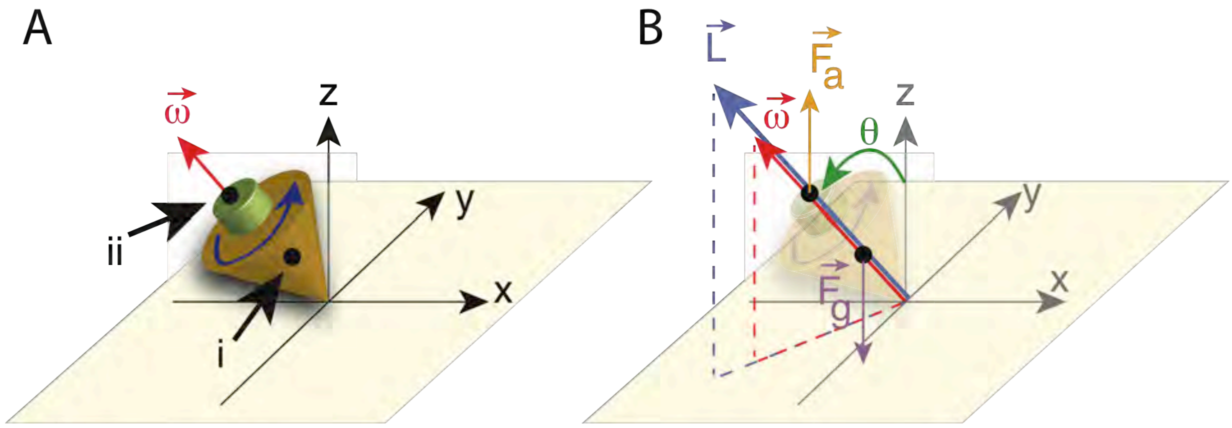

Figure 1. Spinning top with two forces acting on it. (A) A top with mass *M* is spinning with angular velocity $\vec{\omega}$. It has two special points, labeled 'i' and 'ii'. Point 'i' is at a radial distance r_1 from the origin (point of pivot of the spinning top) and point 'ii' is at a radial distance r_2 from the pivot point. That is, the position vector for point 'i' has length r_1 and the position vector for point 'ii' has length r_2. **(B)** The force of gravity $\vec{F_g}$ acts downwards (in the -z direction) at point 'i' at all times. A constant upward force $\vec{F_a}$ (in the +z direction) acts at point 'ii' at all times. The axis of the top is inclined at an angle θ.

(a) Derive, step-by-step, the spinning top's angular speed of precession. Indicate the direction of precession.

(b) What happens to the top when $\vec{F_g} = -\vec{F_a}$?

Problem 2. Entropy of an ideal gas inside a box with two rooms

[*10 points*]

Suppose that a box initially has two rooms inside it (Fig. 2). Both rooms have the same volume. Each room has volume *V*. The box has a total of *N* identical gas particles. The box is surrounded by a thick insulating wall. This means that no energy or particle can enter of leave the box. The total energy in the box thus remains *E* at all times.

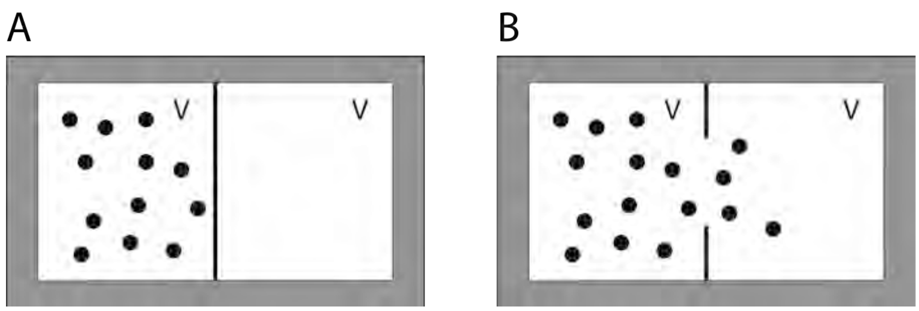

Figure 2. (A) A box with two rooms inside it. A wall divides the two rooms. Each room has volume *V*. One room has *N* idea gas particles. The other room does not have any particles. The impermeable wall does not let any particles to go to the other room. **(B)** The same box as in (A) but now the wall has suddenly opened up. This large hole in the wall allows the particles to move between the two rooms. We assume that the wall between the two rooms has nearly zero volume. In both (A) and (B), a thick wall (thick grey outline) insulates the box from the outside world. Thus no energy or particles can enter or leave the box. Thus the box is an isolated (closed) system.

(a) For the box shown in Fig. 2A, derive the entropy of the system step-by-step. Carefully explain each key step of your derivation.

(b) Now suppose the partition (door) that separates the two rooms opens up (Fig. 2B). After a long time, the gas particles occupy both rooms. All particles can go back in and out of each room. What is the *change* in the entropy, compared to the entropy that you calculated in (a)? You can use the formula for entropy of *N* particles with energy *E* in a box of volume *V* that we derived in class:

$$S = k_B N ln(V) + \frac{3N}{2} k_B ln\left(\frac{2E}{m}\right) + k_B ln(constant)$$

Note that here, we're using the fact that *N* is very large so that $\frac{3N-1}{2} \approx \frac{3N}{2}$.

Note from your answer that the change in entropy is a positive number. This shows that the total entropy has increased as a result of opening the partition between the two rooms. Note that the increase in entropy that you calculate here corresponds to an increase in the *amount of disorder* or equivalently the *amount of information I*. This makes sense. Before we opened the partition between the two rooms, we knew that all the particles were in the left room (Fig. 2A). But after the partition opened, we no longer know which room each particle is in. This corresponds to an increase in the number of *bits* that we now need to specify the system. This corresponds to an increase in the unpredictability of the system, which we showed in the lecture to be a measure of information content of the box.

Problem 3. Entropy of a 2-dimensional gas

[*10 points*]

Derive the entropy of an ideal gas of *N* particles that live in a 2-dimensional cage. That is, each particle can move in x-direction and y-direction but cannot move in the z-direction (i.e., the particles are constrained to move in a plane). The cage has area *A* and it is thermally isolated from its surrounding. This means that no particle can enter or leave the cage and no energy can enter or leave the cage. You can assume that the total energy of the cage of particles is *E*. In class we derived the entropy of an ideal gas that lives inside a 3-dimensional box. Here you're being asked to go through the derivation from the lecture step-by-step, but now modify some of the steps to describe particles living in a 2-dimensional world.

How is your result different from the entropy for a 3-dimensional ideal gas?

Such a system exists in the real world. You can constrain particles (e.g., electrons) to travel on a surface of a material but not inside the bulk of the material. This idea forms the basis of many modern electronic devices.