Physics 1A – Problem set 8

Due: 8:45 AM, Tuesday 26th January, 2016

Problem 1: Warm-up exercises

(a) Given $pV^\gamma =$ constant, prove that $TV^{\gamma-1} =$ constant.
(b) From $W = -\int p\, dV$, show that the work done on the gas in an adiabatic process is given by $W = \frac{p_2V_2 - P_1V_1}{\gamma - 1}$.
(c) The relation $C_p = C_V + R$ holds for any kind of ideal gas (monatomic, diatomic or triatomic). From the equipartition theorem, determine C_V for diatomic and triatomic gases. What is γ in these two cases?
(d) Show that a triatomic gas has a 6 degrees of freedom. What is the average energy of a triatomic molecule?

Problem 2: Expansion of an ideal gas

Consider a monatomic ideal gas consisting of N atoms in an initial volume V_0 at an initial temperature T_0. The gas is then slowly expanded to a final volume $V_1 = 7V_0$. Calculate the temperature T_1, pressure P_1, the work W done by the gas, and the added heat Q, in case the expansion is

(a) Isothermal
(b) Isobaric
(c) Adiabatic

Compare and comment on the results.

Problem 3: The Otto cycle

The Otto cycle describes the working of internal combustion engines such as found in cars, lorries, and electrical generators. The cycle consists of the following processes (see figure next page):

1. First, the gas is adiabatically compressed from initial volume $V_1$to volume V_2.
2. The gas is heated up by adding heat Q_1while the volume is kept constant ($V_2 = V_3$).
3. Then the gas is expanded adiabatically to its initial volume ($V_4 = V_1$).
4. The gas is cooled down at constant volume as heat Q_2 is extracted from the gas.

(A realistic engine based on the Otto cycle contains two more processes but they will be neglected since they cancel each other out and hence do not contribute to any net changes).

(a) What is the efficiency of the engine in terms of the net work done W and/or the heat changes Q_1, Q_2 (you may not need all variables)?
(b) Compute the work done on the gas as (i) it is compressed adiabatically from 2 to 1, (ii) expanded adiabatically from 3 to 4.
(c) Show that the efficiency of the Otto cycle is $1 - r^{1-\gamma}$, where $r = \frac{V_1}{V_2}$ is the compression ratio.

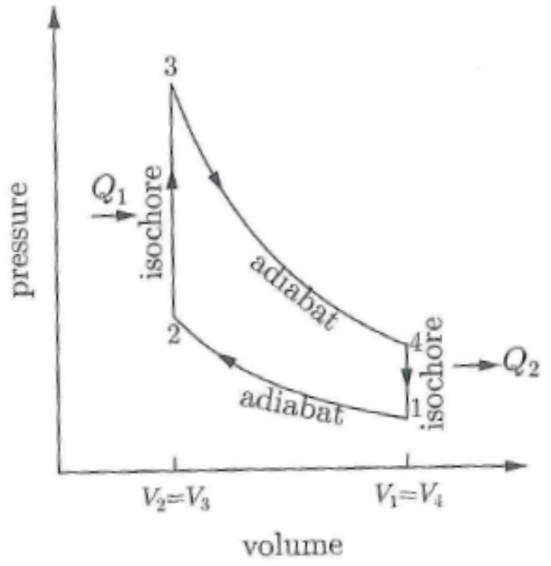

Fig. 13.12 The Otto cycle. (An isochore is a line of constant volume.)

Problem 4: Cooling down

Consider a heat engine which takes a metal bar as high-temperature reservoir and the ocean as low-temperature reservoir. The initial temperature of the metal bar is T_i, the heat capacity of the metal bar is C and is independent of temperature. The ocean temperature is fixed at $T_0 < T_i$. Compute the maximal amount of work the heat engine can deliver, as it cools down from T_i to T_0.