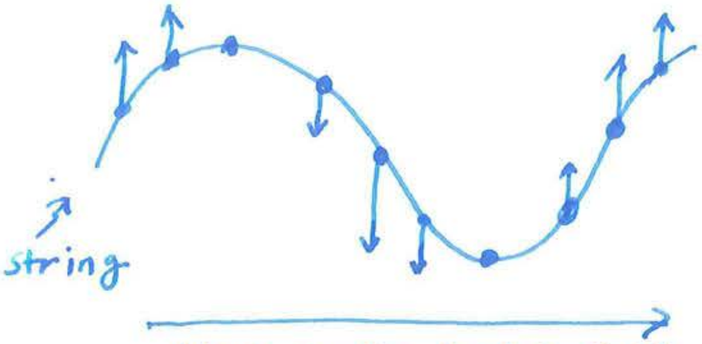
Lecture 8 : Waves (in 1-dimension)

By Hyun Youk
2-Dec.-2015

A Wave = Many (infinite #) simple harmonic oscillators joined together.

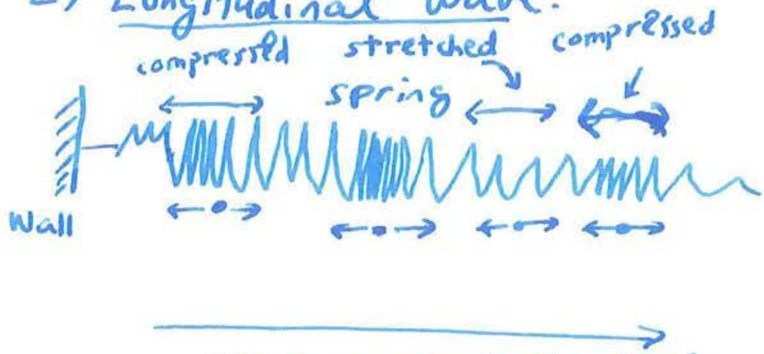
There are 2 types of waves

1.) Transverse wave



Wave moves to the right but each point on the string moves vertically up/down.

2) Longitudinal wave.

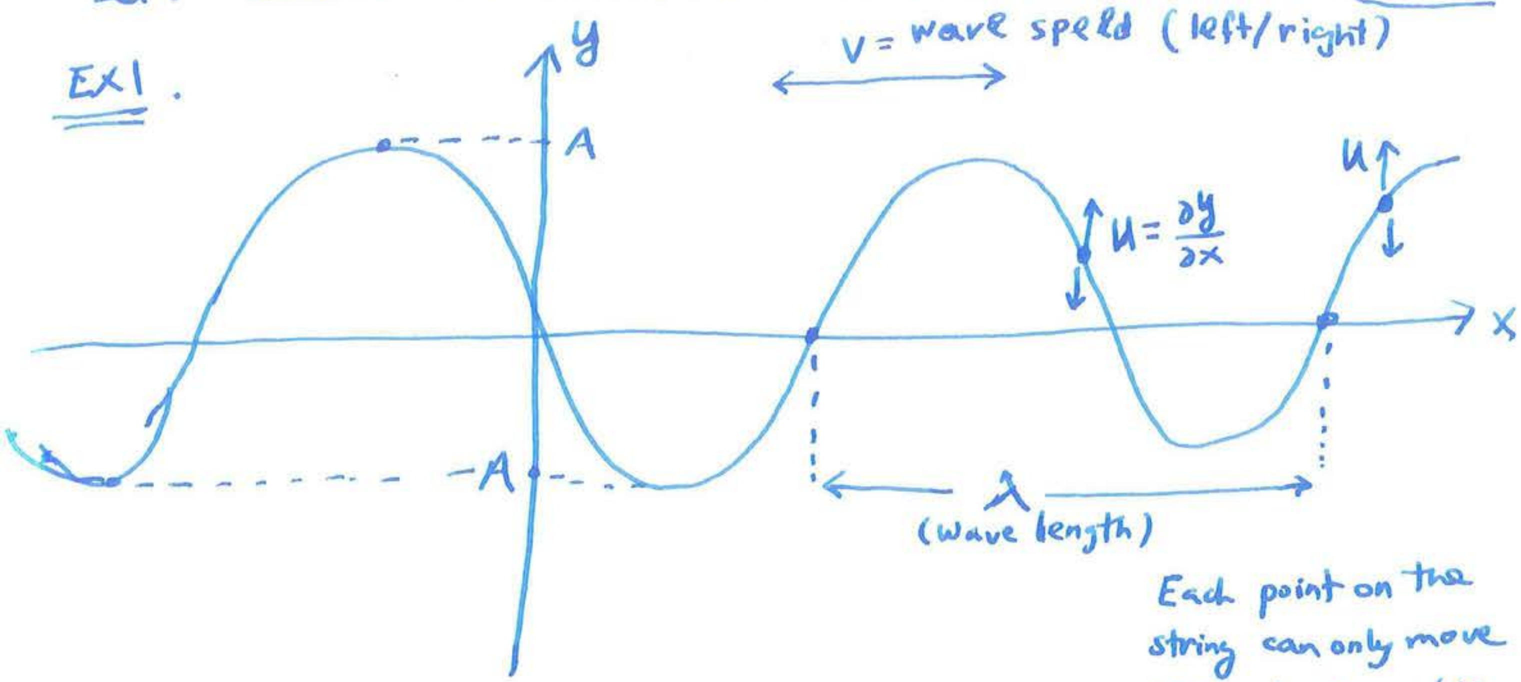


Wave moves to the right
Each point on the spring moves left/right creating compressed & stretched regions periodically.

- Both types of waves are described by the same equation of motion called the "Wave equation".
- Solving this wave equation, you get the ~~~~ position of each point on the string/spring as a function of ~~~~ each point's position and time.

Let's consider a transverse wave.

__EX1.__



Each point on the string can only move up and down. (Not to the right/left)

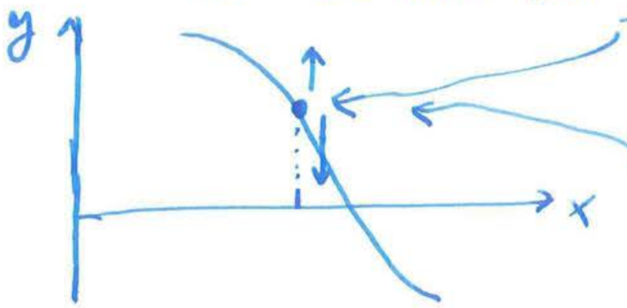
__Question__: What is $y(x,t) = ?$

__Solution__: Let's use the picture above and our mental picture to deduce $y(x,t)$.

$y(x,t) =$ y-position (height) of a point at x, at time t.

The idea here is that at each x, there's a point on the string that moves up/down [so in the y-direction] as a simple harmonic oscillator.

- First, let's keep x constant (i.e. fix the value of x)
  This means that we focus on one particular point (at some x) and we'll see how that point moves up/down.



It's a simple harmonic oscillator so:

$$y_x(t) = A\cos(\omega t + \phi_x)$$

& y-position of this particular point at particular value of x.

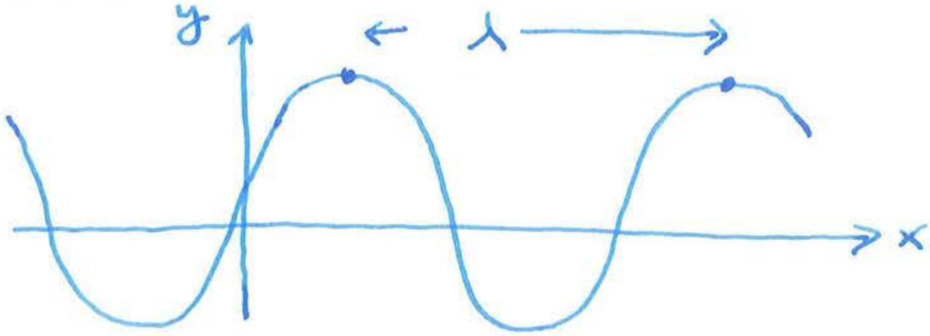
(from last lecture) on simple harmonic motion.

$$y_x(t) = A\cos(\omega t + \phi_x)$$

- ϕ_x: Phase of the SHO at position x
- ω: Angular frequency of SHO
- A: Amplitude
- y_x: This subscript "x" is just saying that this "y" is for a particular point particle at location x.

Now, let us deduce ϕ_x. To do so, let's now take a <u>snapshot</u> at some time t. (i.e. Take a picture of the wave at some particular time t.)

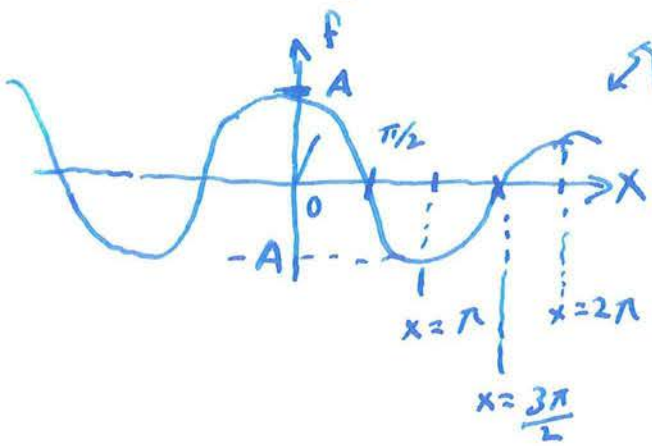
<u>Snapshot picture</u>:



$\lambda \equiv$ wave length.

$\phi_x = \phi(x)$ ← Just another way of writing "ϕ_x"
(ϕ is a function of x.)

Before we go on, let's consider a simple example:



~~f(x)=Acos(x)~~
$f(x) = A\cos(x)$

x in radians

$x = \pi/2$ (radians) $= 90^\circ$

$x = \pi$ (radians) $= 180^\circ$

$x = \frac{3\pi}{2}$ (radians) $= 270^\circ$

$x = 2\pi = \frac{4\pi}{2}$ $= 360^\circ$

Now, what does $g(x) = A\cos\left(\frac{2\pi}{\lambda}\cdot x\right)$ look like?

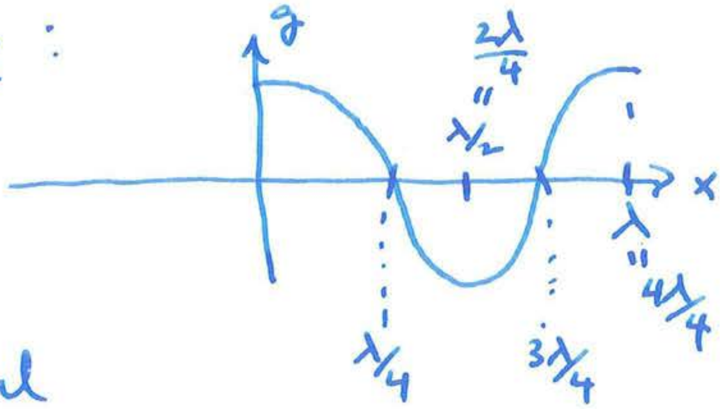
Note that:

- $x = \lambda$: $g(\lambda) = A\cos\left(\frac{2\pi}{\lambda}\cdot\lambda\right)$
$= A\cos(2\pi)$
$= A$

- $x = \frac{\lambda}{2}$ $\left(= \frac{2\lambda}{4}\right)$: $g(\lambda/2) = A\cos\left(\frac{2\pi}{\lambda}\frac{\lambda}{2}\right)$
$= A\cos(\pi)$
$= -A$

- $x = 0$ $\left(= \frac{0\cdot\lambda}{4}\right)$: $g(0) = A$

So graph looks like this:



So going back to our original problem, we can now see that

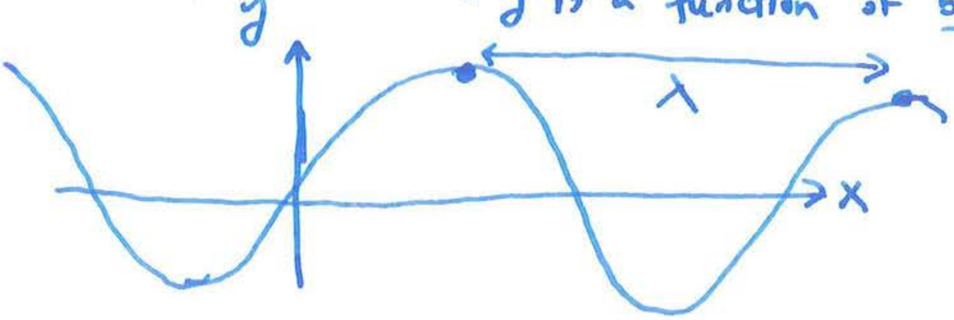
$$\phi_x = \phi(x) = \frac{2\pi}{\lambda}\cdot x$$

So:

$$y_x(t) = A\cos(\omega t + \phi_x)$$

$$\Rightarrow \quad y(x,t) = A\cos\left(\omega t + \frac{2\pi}{\lambda}x\right)$$

& y is a function of both t and x.

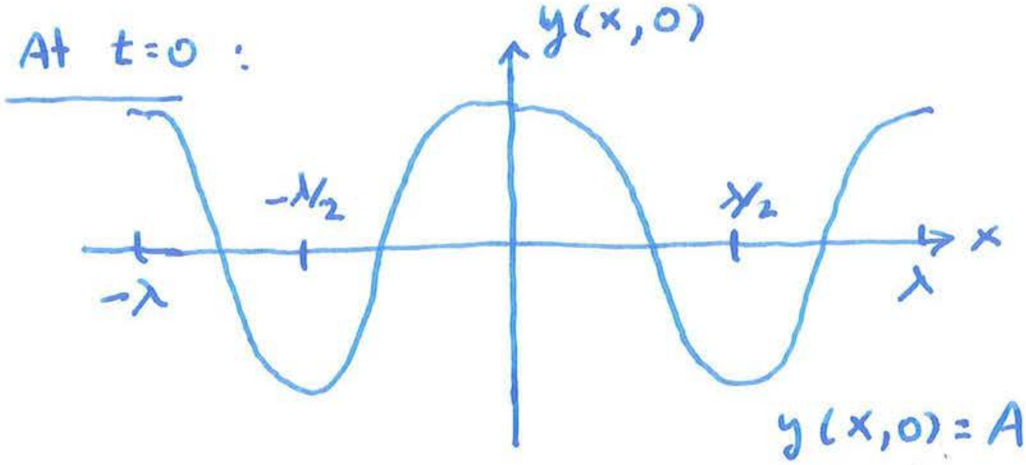


← snapshot at fixed t. If you go over by distance λ, you should see the same point oscillator.

$$y(x,t) = A\cos\left(\omega t + \frac{2\pi x}{\lambda}\right)$$

Let's look at this equation graphically.

At $t=0$:



$$y(x,0) = A\cos\left(\frac{2\pi x}{\lambda}\right)$$

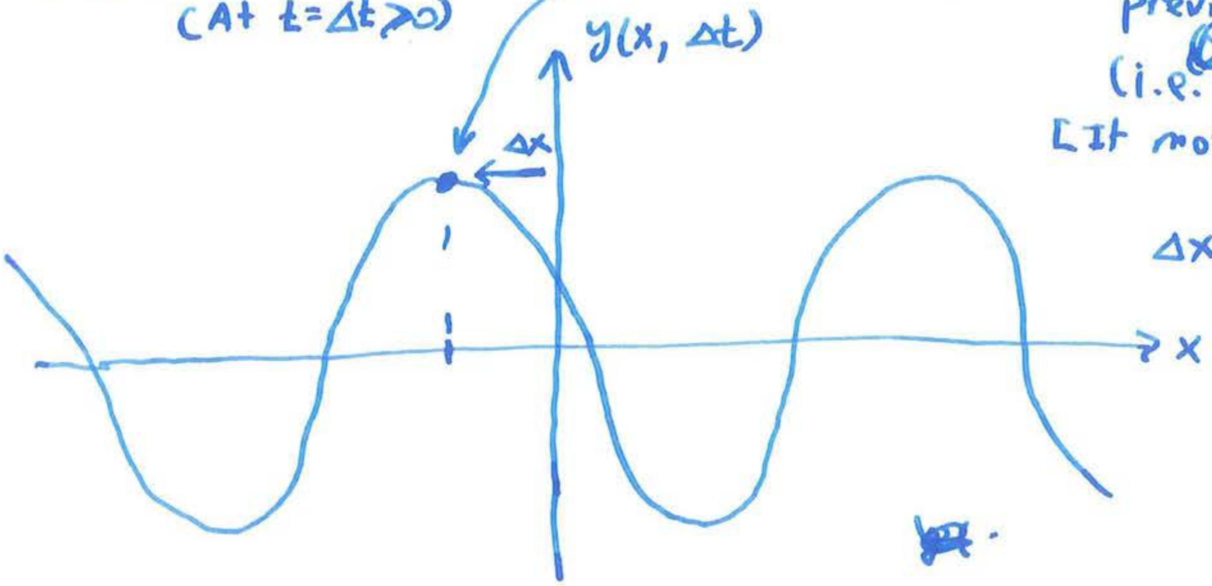
At $t>0$:

($At\ t=\Delta t > 0$)

This point ($y=A$) was at $x=0$ previously (i.e. At $t=0$)

[It moved to the left]

Δx = displacement < 0



$$y(x,\Delta t) = A\cos\left(\omega\Delta t + \frac{2\pi x}{\lambda}\right)$$

$\Delta x < 0$

$$y(\Delta x, \Delta t) = y(0,0)$$

$$A\cos\left(\omega\Delta t + \frac{2\pi}{\lambda}\Delta x\right) = A\cos(0)$$

so:

$$\omega\Delta t + \frac{2\pi}{\lambda}\Delta x = 0.$$

$$\Rightarrow \quad \omega\Delta t = -\frac{2\pi}{\lambda}\Delta x$$

$$\Rightarrow \quad -\frac{\lambda\omega}{2\pi} = \frac{\Delta x}{\Delta t} = \text{wave velocity.}$$

$$\left|\frac{-\lambda\omega}{2\pi}\right| = \left|\text{wave velocity}\right|$$

= wave speed

$= V > 0$

we often ~~write~~ define $\boxed{K = \frac{2\pi}{\lambda}}$

wave number

so it's easier to write.

So: $\boxed{V = \frac{\omega}{k}}$ ← wave speed

(⊕ #).

we just discovered that:

$y(x,t) = A\cos(\omega t + kx)$ is a wave of wave length λ

amplitude A, moving towards the ($\lambda = 2\pi/k$)

negative x direction

(to the left).

what about $y(x,t) = A\cos(kx - \omega t)$?

⊖ sign.

Answer: Again, check what happens to the point at $x=0$.

$y(\Delta x, \Delta t) = y(0,0)$

$A\cos(k\Delta x - \omega\Delta t) = A\cos(0)$

$\Rightarrow \quad k\Delta x - \omega \Delta t = 0$

$\Rightarrow \quad \frac{\Delta x}{\Delta t} = \frac{\omega}{k}$

⊕ number. ⇒ wave moving to the right

(towards ⊕ axis)

⇒

Conclusion:

$$\boxed{\begin{aligned} y(x,t) &= A\cos(kx - \omega t) \\ &\text{or} \\ y(x,t) &= A\cos(kx + \omega t) \end{aligned}}$$

← wave moving to right (⊕ direction)

← wave moving to left (⊖ direction)

solutions describing waves

(solutions to the "wave equation")

~~(Alre Vati~~

In the book (we'll not derive it here.)

- Period of the wave : T

= time required for 1 wavelength to pass by a given location x.

so:

$$\omega T = 2\pi \Rightarrow \boxed{T = \frac{2\pi}{\omega}}$$

period of wave

- frequency of wave : f. = # of cycles / time.

1 cycle (i.e. 1 wave) / period.

$$\Rightarrow \boxed{f = \frac{1}{T}}$$

Wave speed $= V$

- 1 wavelength distance / period.

$\Rightarrow$ $\boxed{V = \frac{\lambda}{T}}$

And since $f = \frac{1}{T}$,
we also have $\boxed{V = \lambda f}$