MITES 2010: Physics III – Survey of Modern Physics
Problem Set 2 – Classical Mechanics: Equations of motion
(Due date: Wed. July 7, 2010 – 10 PM – Mobolaji's Dorm Room)

Problem 1. What a drag!

Suppose a particle moves in one-dimension (i.e., along x-axis). *x(t)* is the particle's position at time *t*. A *drag force* proportional to the particle's velocity *v* acts on it (i.e. Drag force on the particle is $f_{drag} = -bv$). Write down the equation of the motion of the particle. Then by solving this equation, derive the velocity *v(t)* and the position *x(t)* of the particle. If there are any *free parameters* in your solution, describe what they *physically* mean. (If you get stuck on an integration, you can look up the formula in a book / web).

Problem 2. Optical tweezer

Optical tweezer is a device that traps dielectric microspheres using a finely focused laser beam. By sending a beam with just the right intensity profile, the microsphere (bead) can be trapped within a simple harmonic potential well formed by the laser beam, with the center of the beam being the minimum of the potential well. That is, if the bead is displaced by *x* from the trap center, a restoring force of $-k_{trap}x$ acts on the bead in an attempt to return it to the trap center (*x*=*0*). We can also attach polymers such as DNA to the bead, with one end of the DNA tethered to a stationary glass slide. As long as we are dealing with small stretch and compression of DNA, we can model DNA as a Hookian spring-like object with an "effective" spring constant k_{DNA}. Thus, the system shown in Fig. 1(b) can be modeled as a bead of mass *m* attached to two springs (with spring constants k_{DNA} and k_{trap}) as shown in Fig. 1(c).

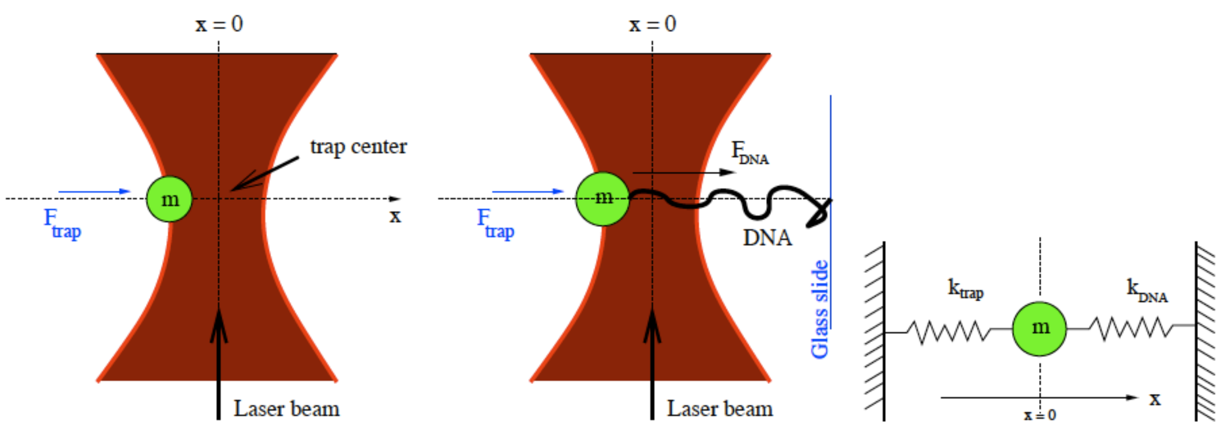

Figure 1.: (a) Left. (b) Middle. (c) Right.: (a) Optical tweezer exerts a restoring force ($-k_{trap}x$) on the microsphere (bead). (b). One end of the DNA is attached to the bead while its other end is tethered to a staionary glass slide. DNA also exerts a restoring force ($-k_{DNA}x$) on the bead. (c) The bead in (b) acts like a block of mass *m* attached to two springs, with spring constants k_{DNA} and k_{trap}.

(a) Derive the equation of motion for the bead (shown in Fig. 1c) and thus show that the bead acts as a simple harmonic oscillator. State what the angular frequency ω is, in terms of k_{DNA}, k_{trap} and *m*. You can assume that *x*=0 is the equilibrium position of the bead and that both springs are at their rest lengths at *x* = 0.

(b) Derive the *general solution* to the equation of motion you derived in (a). Define the physical meaning of all the *free parameters* in your solution.

(c) Write down the total energy of the system shown in Fig. 1(c). At what value(s) of *x* is the kinetic energy maximum? At what value(s) of *x* is the potential energy of "DNA spring" maximum? At what value(s) of *x* is the potential energy of the "trap" maximum? Answer in terms of one of the free parameters you defined in (b). Also, what is the ratio of potential energy stored in the "trap-spring" to the potential energy stored in the "DNA spring" at time *t*?

(d) Suppose that at *t*=0, the bead is at *x*(t=0) =0, and that its velocity at that moment is v_0. By finding specific values for the free parameters in your solution *x(t)* found in (b), write down *x(t)* that describes what the position of the bead is for subsequent times (t > 0).

Problem 3. Normal modes I

Three identical springs and two masses, *m* ande 2*m*, lie between two walls as shown in Fig. 2. Find the normal modes and write down the *general solution* describing *any* arbitrary motion that is executed by the two particles.

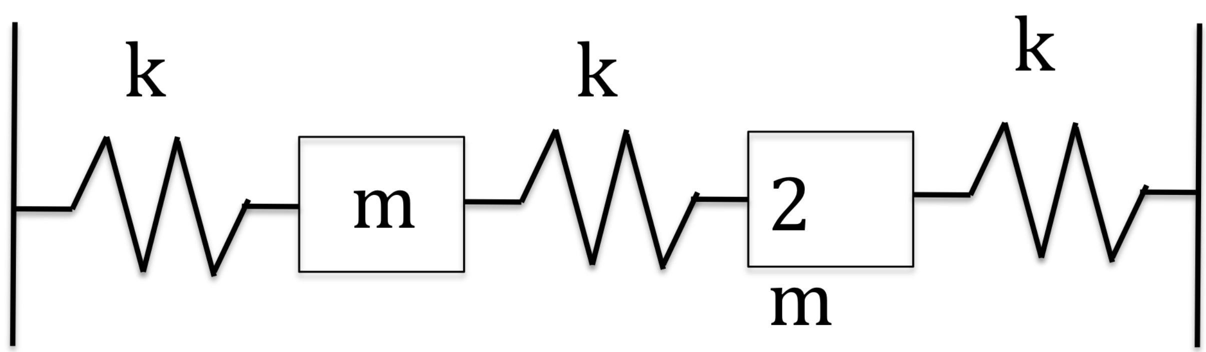

Figure 2

Problem 4. Heading to zero

A particle moves toward *x* = 0 under the influence of a potential $V(x) = -A|x|^n$, where *A* > 0 and *n*>0. The particle has barely enough energy to reach *x* = 0. For what values of *n* will it reach *x=0* in a finite time?

Problem 5. Hanging mass

The potential energy for a mass hanging from a spring is $V(y) = \frac{ky^2}{2} + mgy$, where *y*=0 corresponds to the position of the spring when nothing is hanging from it. Find the frequency of small oscillations around the equilibrium point.

Problem 6. Small oscillations

A particle moves under the influence of the potential $V(y) = -Cx^n e^{-ax}$. Find the frequency of small oscillations around the equilibrium point.

Problem 7. Bead on a rotating hoop

A bead is free to slide along a frictionless hoop of radius *R*. The hoop rotates with constant angular speed ω around a vertical diameter (See Fig. 3). Find the equation of motion for the angle θ shown (angle is in radians). What are the equilibrium positions? What is the frequency of small oscillations about the stable equilibrium? There is one value of ω that is quite special. What is it and why is it special?

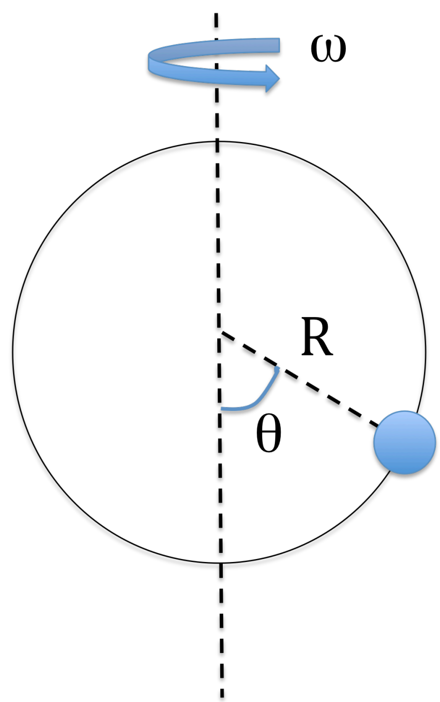

Figure 3

Problem 8. Moving plane

A block of mass m is held motionless on a frictionless plane of mass M and angle of inclination θ (in radians) [Figure 4]. The plane rests on a frictionless horizontal surface. The block is released. What is the horizontal acceleration of the plane? Use Lagrangian to solve this (NOT F=ma). (It turns out you can use F = ma to solve this problem, but this turns out to be a much more difficult way to solve this problem).

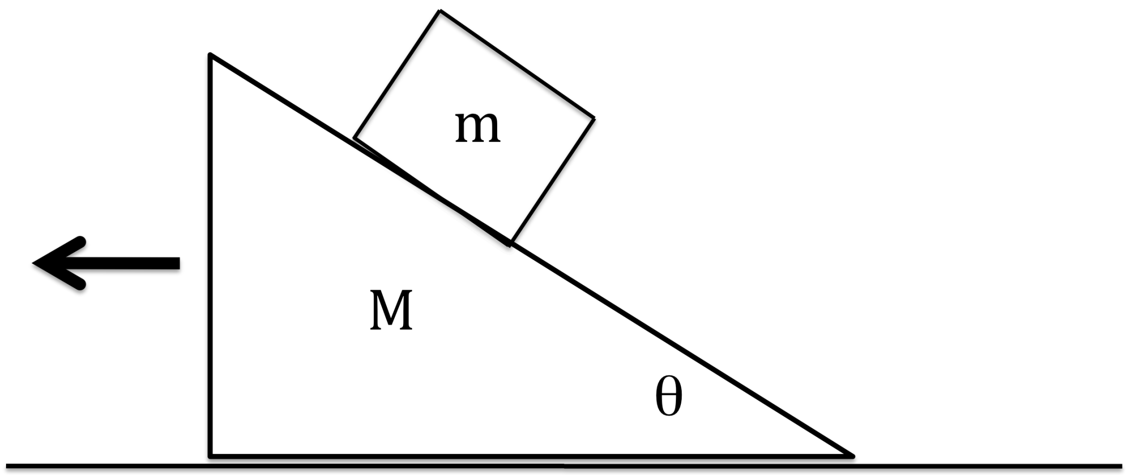

Figure 4