Hints on Physics III: Problem set #4

Hint on Problem 2: This question is asking you to start with the definition of impedance: $Z = \frac{F_0}{v_0}$, then show that it's equal to ρc, where c is the wave speed. Notice that v_0 is the maximum speed that a particle located at a particular position x simple harmonically oscillates vertically can have. You can find out what this is by starting from $y(x,t) = Asin(kx - \omega t)$, given to you in the problem. Similarly, you can also find what F_0 (maximal force applied to the particle at position x) from $y(x,t)$ and T. Notice that when θ is small (i.e. $|\theta| \ll 1$), then $sin(\theta) \approx \frac{sin(\theta)}{cos(\theta)} = tan(\theta)$, since $\cos(\theta) \approx 1$. You will need to use the relations we derived in class (which you should be able to derive yourself within seconds): such as $kc = \omega$, and for a string, $c = \sqrt{\frac{T}{\rho}}$

Hint on Problem 3: The hardest part of this problem (and its analogue, problem 5) is figuring out what the boundary condition applied at the junction between two different strings must be. In this problem, the junction joining the two strings is at $x = 0$. As I eluded to in the problem, there are two boundary conditions you need to come up with. Physically, the boundary conditions are as follows:
1. The two strings are joined to each other at $x = 0$.
2. There is no kink at the joint ($x = 0$). This means that the junction is smooth.

If you say $y_1(x,t)$ and $y_2(x,t)$ describe the shape of string A and string B respectively, then above conditions reduce to

1. $y_1(0,t) = y_2(0,t)$ – (Two strings are joined to each other at $x = 0$)
2. $\left.\frac{\partial y_1}{\partial x}\right|_{x=0} = \left.\frac{\partial y_2}{\partial x}\right|_{x=0}$ (at all time t) – (Smooth junction, no kinks)

So then what are $y_1(x,t)$ and $y_2(x,t)$? From the diagram (and thinking about waves moving in the two strings), $y_1(x,t)$ is a **resultant wave** consisting of a superposition of incident and reflected waves. This resultant wave is what you'll see with your eyes in an experiment, NOT the composite waves (incident and reflected) making up the resultant wave. $y_2(x,t)$ is just the transmitted wave in string B. Notice that in this problem, we are working with complex numbers so both $y_1(x,t)$ and $y_2(x,t)$ are complex-valued functions.

Recall from class notes that a complex number representation of a plane wave is $Aexp(i(kx - \omega t))$ if a wave is moving to the right, and $Aexp(i(-kx - \omega t))$ if the plane wave is moving to the left. These are also given directly to you on the Pset handout. A question you need to ask yourself is if the wave number k and angular frequency ω change as the wave moves from one medium (string) to another. To reason this out physically, notice that $k = \frac{2\pi}{\lambda}$, and $\omega = kc$. Both are quantities we derived in class (c = wave speed).

Suppose you hold the left end of string A, and wiggle your hand simple harmonically up and down with angular frequency ω_0. This results in generation of incident wave of angular frequency ω. Can you figure out what the angular frequency of reflected and transmitted waves must be? Think of what would happen if the transmitted wave in string B has angular frequency ω which does NOT equal ω_0. In that case, you'll have waves "piling up" at the junction, which is not physical. So ω must equal ω_0. What about the angular frequency of the reflected wave? By same reasoning, it must also be traveling with angular frequency ω_0.

What about k? Is it different depending on which string the wave is traveling in? Answer is that it is. You may physically reason through this by thinking of what would happen if string B is more "dense" medium. So it would be harder to generate a wave. But $kc = \omega$ and we already reasoned that ω is the same for both strings. But c is clearly different for both strings (different wave speeds). Thus, k must be different for both strings. (i.e. the incident, reflected waves travel with wavelength λ_1, while the transmitted wave travel with wavelength λ_2).

Hint on Problem 5.: This is just like problem 3 except that you now have two boundaries, because there are two junctions joining different strings. Apply the same boundary conditions but at $x = 0$ and $x = L$.