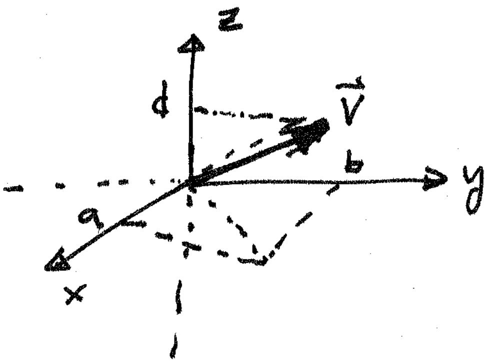
Wed. July 16-08

Just to recap what Fourier series is about:

Useful analogy to dot products of vectors:

Vectors in 3D.



Any vector $\vec{V}$ in 3D can be written as:

$$\vec{V} = a\hat{x} + b\hat{y} + d\hat{z}$$

Goal: find $a = ?$ $b = ?$ $d = ?$

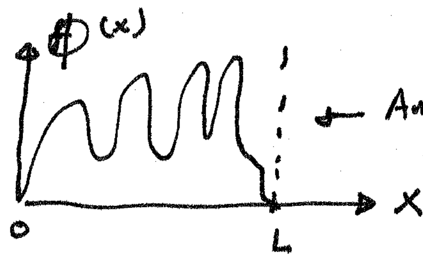
Dot product properties:

$$\hat{x}\cdot\hat{y} = 0$$
$$\hat{x}\cdot\hat{z} = 0$$
$$\hat{x}\cdot\hat{x} = 1$$
$$\hat{y}\cdot\hat{z} = 0$$
$$\hat{y}\cdot\hat{y} = 1$$
$$\hat{z}\cdot\hat{z} = 1$$

Extract coefficient a, b, c.

$$a = \vec{V}\cdot\hat{x}$$
$$b = \vec{V}\cdot\hat{y}$$
$$d = \vec{V}\cdot\hat{z}$$

Fourier series



← Any function $\phi(x)$ can be written as sum of $\sin\left(\frac{n\pi x}{L}\right)$ $n = 1, 2, 3, 4, \ldots$

Consider particular $\phi(x)$ that can be written as following: ($A_4 = 0$, $A_5 = 0$, … $A_n = 0$ $n \geq 4$)

$$\phi(x) = A_1 \sin\left(\frac{\pi x}{L}\right) + A_2 \sin\left(\frac{2\pi x}{L}\right) + A_3 \sin\left(\frac{3\pi x}{L}\right)$$

Goal: find $A_1 = ?$ $A_2 = ?$ $A_3 = ?$

Integration property:

$$\int_0^L \sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi x}{L}\right)dx = 0$$

$$\int_0^L \sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{3\pi x}{L}\right)dx = 0$$

$$\int_0^L \sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{\pi x}{L}\right)dx = \frac{L}{2}$$

$$\Rightarrow \frac{2}{L}\int_0^L \sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{\pi x}{L}\right)dx = 1$$

$$A_1 = \frac{2}{L}\int_0^L \phi(x)\sin\left(\frac{\pi x}{L}\right)dx$$

$$A_2 = \frac{2}{L}\int_0^L \phi(x)\sin\left(\frac{2\pi x}{L}\right)dx$$

$$A_3 = \frac{2}{L}\int_0^L \phi(x)\sin\left(\frac{3\pi x}{L}\right)dx.$$

Continued

In general, vector $\vec{V}$ in N-dimensional space:

$$\vec{V} = A_1\hat{x}_1 + A_2\hat{x}_2 + A_3\hat{x}_3 + A_4\hat{x}_4 + \dots A_N\hat{x}_N$$

where $\hat{x}_i$ is orthogonal to $\hat{x}_j$ (if $i \neq j$)

$\Rightarrow$ $\hat{x}_i \cdot \hat{x}_j = 0$ $(i \neq j)$

$\hat{x}_i \cdot \hat{x}_i = 1$

Goal: $A_1 = ?$, $A_2 = ?$, $A_3 = ?$ ....

And as before (on previous pg.) extract the coefficients by:

$$A_1 = \vec{V} \cdot \hat{x}_1$$

$$A_2 = \vec{V} \cdot \hat{x}_2$$

$\vdots$

Fourier series (continued)

$$\phi(x) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{L}\right)$$

$(N = \infty)$

$$\frac{2}{L}\int_0^L \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right)dx = 0$$

(if $n \neq m$)

n, m are (+) integers.

$$\frac{2}{L}\int_0^L \sin^2\left(\frac{n\pi x}{L}\right) = 1.$$

Goal: $A_1 = ?$ $A_2 = ?$, ...

As before, extract the coefficients by

$$A_1 = \frac{2}{L}\int_0^L \phi(x)\sin\left(\frac{\pi x}{L}\right)dx$$

$$A_2 = \frac{2}{L}\int_0^L \phi(x)\sin\left(\frac{2\pi x}{L}\right)dx$$

$\vdots$