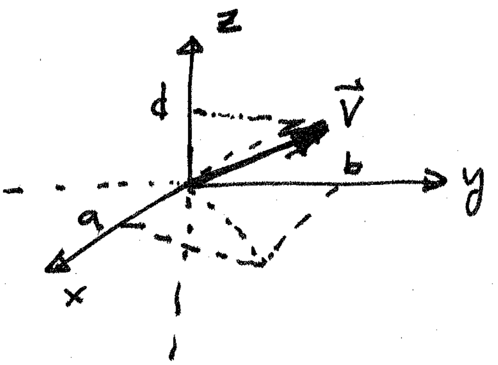
Wed. July 16, 08

Just to recap what Fourier series is about:

Useful analogy to dot products of vectors:

Vectors in 3D. :



Any vector $\vec{V}$ in 3D can be written as:

$$\vec{V} = a\hat{x} + b\hat{y} + d\hat{z}$$

Goal: find $a = ?$ $b = ?$ $d = ?$

Dot product properties:

$\hat{x}\cdot\hat{y} = 0$

$\hat{x}\cdot\hat{z} = 0$

$\hat{x}\cdot\hat{x} = 1$

$\hat{y}\cdot\hat{z} = 0$

$\hat{y}\cdot\hat{y} = 1$

$\hat{z}\cdot\hat{z} = 1$

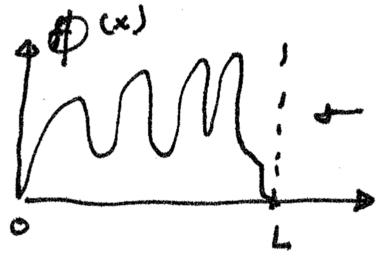
Extract coefficient a, b, c.

$a = \vec{V}\cdot\hat{x}$

$b = \vec{V}\cdot\hat{y}$

$d = \vec{V}\cdot\hat{z}$

Fourier series



Any function $\phi(x)$ can be written as sum of $\sin\left(\frac{n\pi x}{L}\right)$ $n = 1, 2, 3, 4, \ldots$

Consider particular $\phi(x)$ that can be written as following: ($A_4 = 0$, $A_5 = 0$, ... $A_n = 0$ $n \geq 4$)

$$\phi(x) = A_1 \sin\left(\frac{\pi x}{L}\right) + A_2 \sin\left(\frac{2\pi x}{L}\right) + A_3 \sin\left(\frac{3\pi x}{L}\right)$$

Goal: find $A_1 = ?$ $A_2 = ?$ $A_3 = ?$

Integration property:

$$\int_0^L \sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi x}{L}\right)dx = 0$$

$$\int_0^L \sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{3\pi x}{L}\right)dx = 0$$

$$\int_0^L \sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{\pi x}{L}\right)dx = \frac{L}{2}$$

$$\Rightarrow \frac{2}{L}\int_0^L \sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{\pi x}{L}\right)dx = 1$$

$$\begin{cases} A_1 = \frac{2}{L}\int_0^L \phi(x)\sin\left(\frac{\pi x}{L}\right)dx \\ A_2 = \frac{2}{L}\int_0^L \phi(x)\sin\left(\frac{2\pi x}{L}\right)dx \\ A_3 = \frac{2}{L}\int_0^L \phi(x)\sin\left(\frac{3\pi x}{L}\right)dx. \end{cases}$$

In general, vector $\vec{V}$ in N-dimensional space:

Fourier series (continued)

$$\vec{V} = A_1\hat{x}_1 + A_2\hat{x}_2 + A_3\hat{x}_3 + A_4\hat{x}_4 + \dots A_N\hat{x}_N \quad \longleftrightarrow \quad \phi(x) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{L}\right) \quad (N = \infty)$$

where $\hat{x}_i$ is orthogonal to $\hat{x}_j$ (if $i \neq j$)

$\Rightarrow \quad \hat{x}_i \cdot \hat{x}_j = 0 \quad (i \neq j)$

$\hat{x}_i \cdot \hat{x}_i = 1$

$\longleftrightarrow$

$$\frac{2}{L}\int_0^L \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right)dx = 0 \quad (\text{if } n \neq m)$$

n, m are (+) integers.

$$\frac{2}{L}\int_0^L \sin^2\left(\frac{n\pi x}{L}\right) = 1.$$

Goal: $A_1 = ?$, $A_2 = ?$, $A_3 = ?$ .... $\longleftrightarrow$ Goal: $A_1 = ?$ $A_2 = ?$, ...

And as before (on previous pg.) extract the coefficients by:

$$A_1 = \vec{V}\cdot\hat{x}_1$$

$$A_2 = \vec{V}\cdot\hat{x}_2$$

$\vdots$

$\longleftrightarrow$

As before, extract the coefficients by

$$A_1 = \frac{2}{L}\int_0^L \phi(x)\sin\left(\frac{\pi x}{L}\right)dx$$

$$A_2 = \frac{2}{L}\int_0^L \phi(x)\sin\left(\frac{2\pi x}{L}\right)dx$$

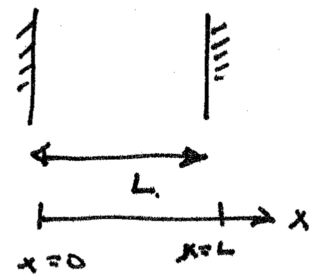
$\vdots$

How can we generalize (extend) the Fourier sine series

$$\phi(x) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{L}\right) \quad ?$$

Remember that L = distance of separation between 2 walls.

Consider what would happen when we push the 2 walls farther and further apart.



$\Rightarrow$ limit of $L \to +\infty$.

Recall that $k_n = \frac{n\pi}{L}$ ← wave # $\left(= \frac{2\pi}{\lambda_n}\right.$; $\lambda_n \equiv$ wavelength of n^{th} normal mode$\left.\right)$ $(n = 1, 2, 3, \ldots)$

so can write:

$$\phi(x) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{L}\right) = \sum_{k_n} A_n \sin(k_n x)$$

(remember that we got these normal modes by finding what standing waves can fit between the 2 walls.)

$$\left(k_1 = \frac{\pi}{L},\ k_2 = \frac{2\pi}{L},\ k_3 = \frac{3\pi}{L}, \ldots\right.$$

and

$$\left.\begin{aligned} k_{j+1} &= \frac{(j+1)\pi}{L} \\ k_j &= \frac{j\pi}{L} \end{aligned}\right\} \Rightarrow k_{j+1} - k_j = \frac{\pi}{L} \ll 1$$

← very small (infinitesimal as $L \to \infty$)

Letting $\boxed{\Delta k = \pi/L}$, we can rewrite the Fourier series as:

$$\begin{aligned} \phi(x) &= \sum_{k_n} A_n \sin(k_n x) \\ &= \frac{1}{\Delta k} \sum_{k_n} \Delta k\, A_n \sin(k_n x) \\ &= \frac{1}{\Delta k} \sum_{k_n} dk\, A_k \sin(k_n x) \\ &= \int_0^{\infty} dk\, \frac{A_k}{\Delta k} \sin(kx) \\ &= \int_0^{\infty} dk\, \tilde{A}_k \sin(kx) \end{aligned}$$

but Δk so small that $\Delta k \equiv dk$ ← infinitesimal quantity.

← And write A_n as A_k.

← k runs from 0 to ∞ since $k_1 = \pi/L \approx 0$ ($\because L \gg \pi$).

Define $\tilde{A}(k) = \frac{A(k)}{\Delta k}$.

Hence, $\phi(x) = \int_0^\infty dk\, \tilde{A}_k \sin(kx)$ when $L \to \infty$.

But instead of writing A_k, we write $A(k)$
(since A is a function of k)

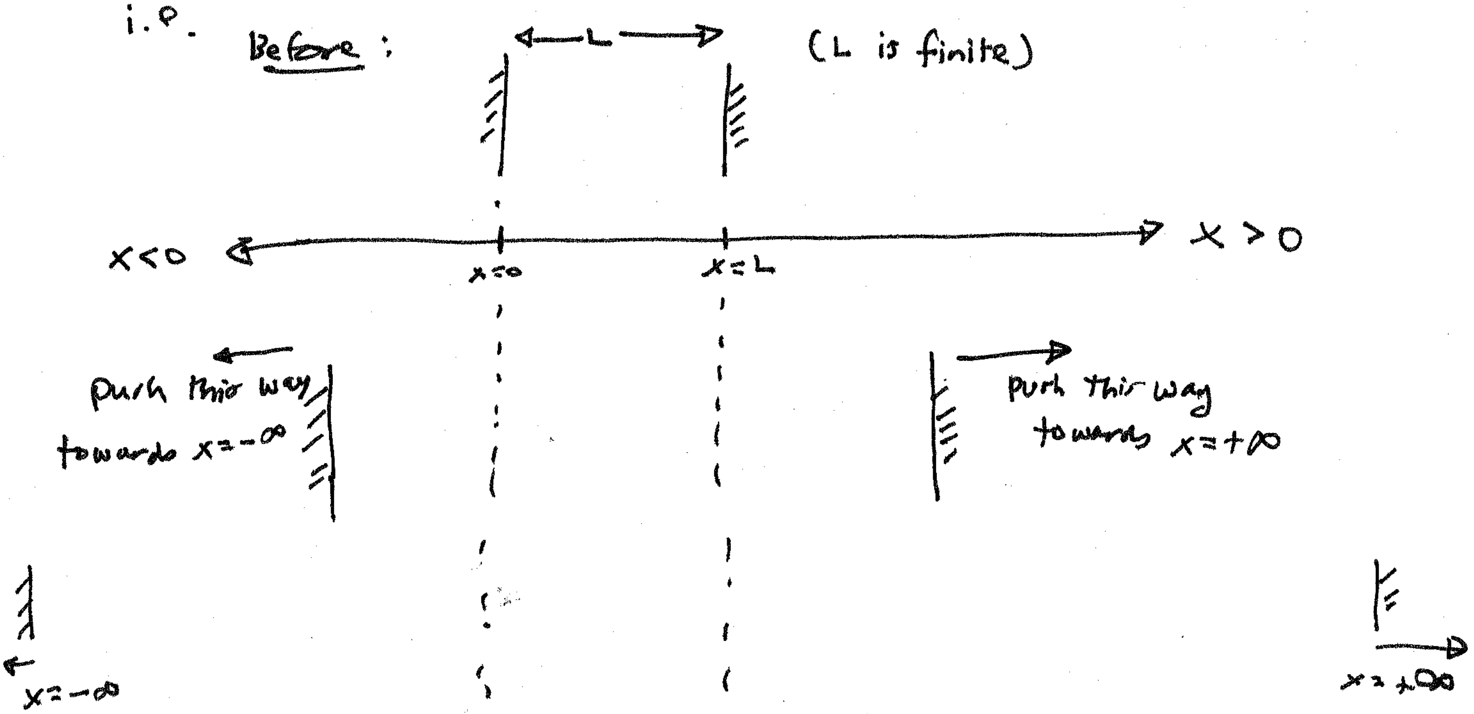
$$\Rightarrow \boxed{\phi(x) = \int_0^\infty dk\, A(k) \sin(kx)}$$

← Note: I just wrote $A(k)$ instead of "$\tilde{A}(k)$" since I changed notation to save time by not writing "~"

Notice that by pushing the 2 walls apart so much that $L \to \infty$, we are able to enclose our entire (1D version) of universe between the 2 walls. ~~that~~ But there's a bit of subtlety involved here. Since the 2 walls are at the ~~ends~~ 2 opposite ends of our 1D universe, we can relax the boundary condition that $\phi(x)$ be zero at both walls.

To see this, notice that the left wall is now at $x = -\infty$ and the right wall is now at $x = +\infty$.

i.e. Before: (L is finite)



So, since the 2 walls are now at $x = \pm\infty$,

and $\phi(x = \pm\infty)$ is not defined anyway (i.e. you can't define any function at $x = +\infty$ and $x = -\infty$.)

we can't even say "$\phi(x = \pm\infty) = 0$"

so there's no boundary condition anymore! when $L \to \infty$.

Recall that when we solved the wave eq'n between 2 finite walls, $(L < \infty)$, we initially guessed a solution of form:

$$\phi(x) = \sum_{n=1}^{\infty} \left\{ A_n \sin\left(\frac{n\pi x}{L}\right) + B_n \cos\left(\frac{n\pi x}{L}\right) \right\}.$$

(and we found that $B_n = 0$ for all n in order to satisfy the boundary conditions $\phi(x=0) = 0$ and $\phi(x=L) = 0$.)

But since we no longer have this constraint, $B_n \neq 0$ anymore.

So: when $L \to \infty$:

$$\phi(x) = \sum_{n=1}^{\infty} \left\{ A_n \sin\left(\frac{n\pi x}{L}\right) + B_n \cos\left(\frac{n\pi x}{L}\right) \right\}$$

And in fact, this is actually an integral (now involving cosine as well) by the same procedure used on pg 109 & pg 110:

$$\Rightarrow \boxed{\phi(x) = \left\{ \int_0^{\infty} dk\, A(k) \sin(kx) + \int_0^{\infty} dk\, B(k) \cos(kx) \right\}}$$

is the general (Real) Fourier series describing any function $\phi(x)$ defined on $x \in (-\infty, +\infty)$

What happens if we have a <u>math</u> (not physics) problem of finding Fourier series representation of a Complex valued function $\phi(x)$ (where $x \in \mathbb{R}$ number) ?
(real number.)

<u>Ans</u> : $$\boxed{\phi(x) = \int_{-\infty}^{\infty} \tilde{A}(k)\, e^{ikx}\, dk}$$ where $\tilde{A}(k)$ is some $\mathbb{C}$-valued function of k.

1 To get this, just take the <u>real</u> - version $\phi(x)$ we wrote down at the bottom of pg 107, and notice that

$$\int_0^{\infty} dk \left\{ A(k) \sin(kx) + B(k) \cos(kx) \right\}$$

$$= \int_0^{\infty} dk \left\{ A(k) \left[\frac{e^{ikx} - e^{-ikx}}{2i} \right] + B(k) \left[\frac{e^{ikx} + e^{-ikx}}{2} \right] \right\}$$

$$= \int_0^{\infty} \left\{ \underbrace{\left(\frac{A(k)}{2i} + \frac{B(k)}{2} \right)} e^{ikx} + \underbrace{\left(\frac{B(k)}{2} - \frac{A(k)}{2i} \right)} e^{-ikx} \right\}$$

$$= \int_0^{\infty} dk \left(\cdots \right) e^{ikx} + \int_0^{\infty} dk \left(\cdots \right) e^{-ikx}$$

$$= \int_0^{\infty} dk \left(\underset{①}{\cdots} \right) e^{ikx} + \int_{-\infty}^{0} dk \left(\underset{②}{\cdots} \right) e^{ikx}$$

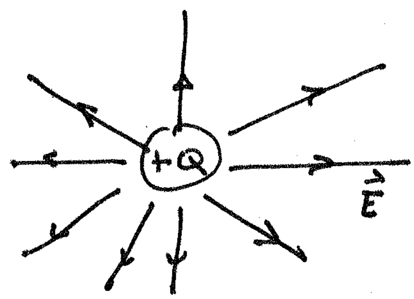
$$= \int_{-\infty}^{\infty} dk\, \tilde{A}(k)\, e^{ikx}$$

$\tilde{A}(k)$ is constant ~~arbitrary~~ $\begin{cases} = ① & \text{if } k > 0 \\ = ② & \text{if } k < 0 \end{cases}$

4. Electromagnetic Waves

Quick Basics of Electricity & Magnetism (E&M):

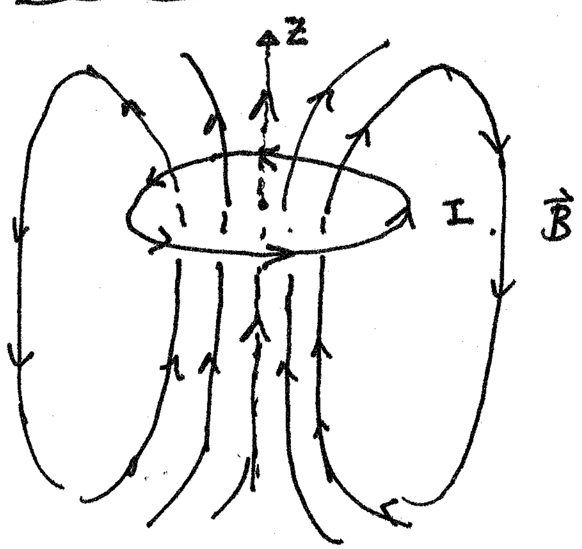
* **Electric field** $\vec{E}$ arises from electric charges.



$\vec{E}$ radially emitted by point charge +Q.

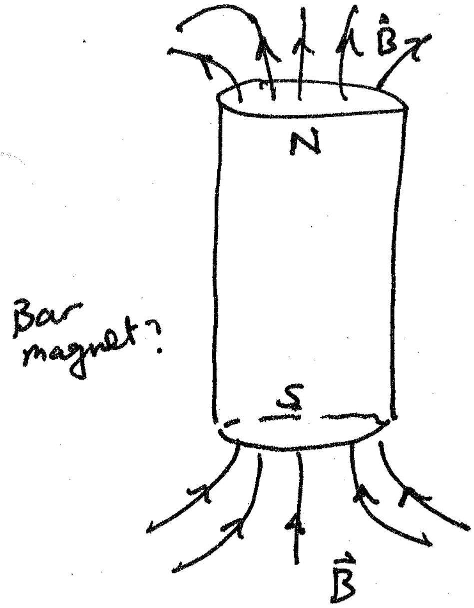
* **Magnetic field** $\vec{B}$ arises from moving charges. (~~there~~ there are no free magnetic charges)

In high school, you probably dealt with steady-state current = I. (i.e. uniform, (over time and space) constant current.)



* Question: How does a bar magnet retain its magnetic properties? That is, how does a bar magnet generate ~~electric~~ magnetic field when there's no electric current going through it?

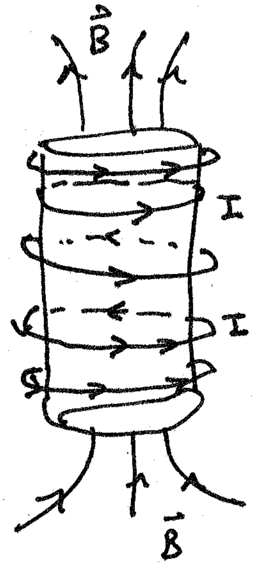
over



← bar magnet (shape of cylinder)

Clearly has no free electric current flowing through/around it. yet it generates magnetic field. How?

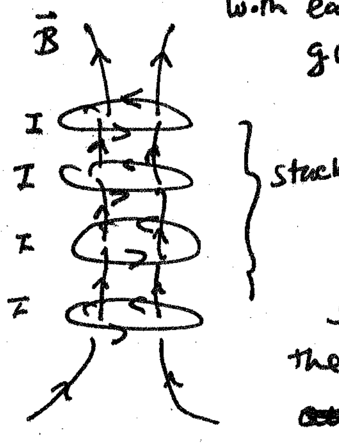
Before answering this question, consider a solenoid with electric current carrying wire wrapped around it.



← current carrying wire wrapped around a cylinder

We can explain how $\vec{B}$ is generated ~~for the pat~~ by the solenoid since the solenoid is just a stack of circular current carrying wires:

with each circular wire generating its own $\vec{B}$



So the net magnetic field $\vec{B}$ generated by the solenoid is just the superposition of $\vec{B}$ generated by each circular current.

Now, going back to explaining how a bar magnet generates its magnetic field despite lacking an apparent electric current, we need to think about structure of atom: <u>Bohr model</u>.

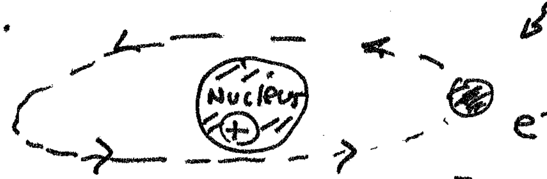
(<u>N.B.</u> To really explain this, we need <u>quantum mechanics</u>. But we're at the moment just interested in how people <u>used</u> to explain this using

~~pre-quan~~ ↙ sentence continued from previous pg.

pre-quantum physics.)
(classical)

Bohr model of ~~a~~ Atom:

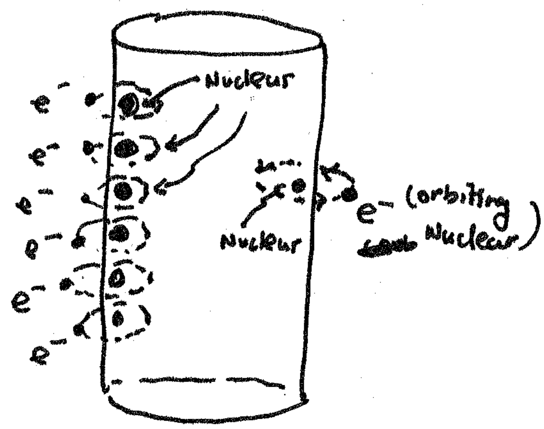
called "bound e^-"



Electron orbits the positively charged nucleus.

(e^- going around in a circular orbit)

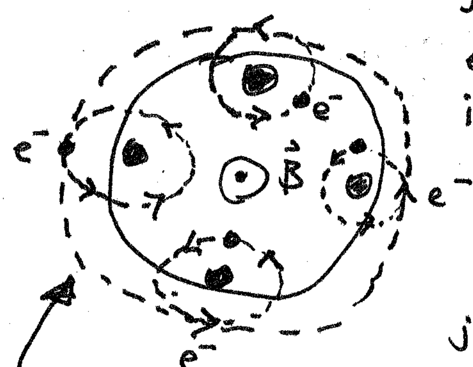
The bar magnet is made up of atoms, and in particular, if we pay attention to those atoms at the edges (boundaries) of the magnet:



Each bound e^- is orbiting the nucleus of atom its bound to.

But looking at those bound e^-'s at the edge of the bar magnet:

Top view of cylinder:

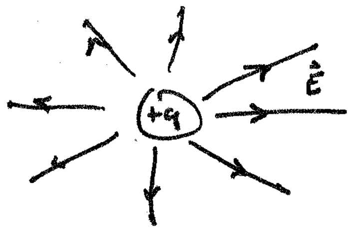


Joining the outer rim of each bound e^-'s orbit, it looks like there's a net circular current wrapped around the cylinder. just like a solenoid.

This "apparent" circular ring of current is called "bound current"

as opposed to "free current" which refers to electric current due to e^-'s not bound to any atom, as in the wires wrapping solenoid.

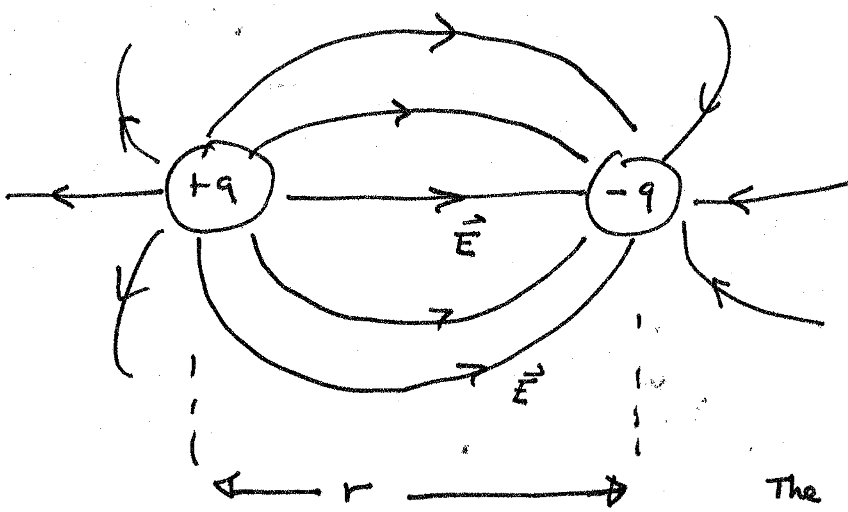
Now that we have some sense of how $\vec{E}$ and $\vec{B}$ are generated, can we convince ourselves that electromagnetic waves must exist?



← These electric fields are NOT waves. Waves move; these $\vec{E}$ due to the stationary charge +q do not move; they are static.

Similarly, magnetic field $\vec{B}$ generated by a stationary bar magnet or solenoid is not a ~~field~~ wave : $\vec{B}$ remains static.

But consider the following situation:



The positive charge exerts an attractive force on −q of $F = \frac{kq^2}{r^2}$

$r \equiv$ distance of separation between the 2 charges.

Now, suppose you grab the +q charge with your hand, and pull it further away from −q. (Thus increasing the distance of separation between 2 charges from r to R.)

Qu: How long does it take for the $-q$ charge to feel that $+q$ has moved? Does $-q$ instantaneously feel the new force or does it take some time to respond to $+q$ having moved?

To answer this, imagine that r is very large. Say r is the distance between Pluto and Earth.
($+q$ is on Pluto) ($-q$ on Earth)

It should make you feel very uneasy if I told you that somehow the $-q$ on Earth immediately, instantaneously, feels that $+q$ on Pluto has moved by some distance. In fact, it takes some time before the $-q$ charge feels the movement of $+q$ on Pluto.

$\Rightarrow$ $+q$ charge sends out a signal to $-q$ in the form of a disturbance in $\vec{E}$ field that already exists between the 2 charges. $\Rightarrow$ This disturbance is electromagnetic wave and it travels at the speed of light.

Now that we've convinced ourselves that EM wave should exist, we need to find out what mathematics describes EM waves.

To do that, we turn to Maxwell's eqns:

It takes about a semester long course to develop and then explain the consequences of Maxwell's eqns, time that we don't have. So we'll just take these equations for granted and derive EM wave eqns from them.

Last time, we stated (without derivation since we don't have the entire semester to do this!)

the **Maxwell's eqns** describing all of classical electricity & magnetism phenomena. (i.e. No quantum stuff.)

Basically, this is the bulk of electromagnetic phenomena we deal with on a day to day basis.

Let's remind ourselves : Maxwell's eqns are: (where $\vec{E}(x,y,z)$ = Electric field, $\vec{B}(x,y,z)$ = Magnetic field.)

① $\nabla \cdot \vec{E} = \rho/\varepsilon$

② $\nabla \cdot \vec{B} = 0$

2 eqns describing divergences of $\vec{E}$ & $\vec{B}$ fields ($\nabla \cdot$)

③ $\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

④ $\nabla \times \vec{B} = \mu \vec{J} + \mu\varepsilon \dfrac{\partial \vec{E}}{\partial t}$

2 eqns describing curl of $\vec{E}$ & $\vec{B}$ ($\nabla\times$).

(ρ = electrical charge density, $\vec{J}$ = Electrical current density, $\mu \equiv$ permeability of free space, $\varepsilon \equiv$ permittivity of free space.)

In vector calculus class, you learn that "∇" (gradient operator) can be considered as a "vector":

$$\boxed{\begin{aligned}\nabla &= \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\\ &= \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\end{aligned}}$$

and so **Divergence**: $\nabla \cdot \vec{E} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\cdot(E_x, E_y, E_z)$

"dot product" between ∇ and $\vec{E}$

$$= \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

and **curl**: $\nabla \times \vec{E} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\times(E_x, E_y, E_z)$

"cross product" between ∇ and $\vec{E}$.

$$= \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}, \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}, \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right).$$

And recall that we derived the Electromagnetic wave eq'ns just by manipulating the Maxwell's eq'ns:

That is: $\nabla \times (\nabla \times \vec{E}) = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E}$ ← from vector calculus, you get this identity.

called "Laplacian"

but in free space (vacuum, such as deep in outer space), there are no electric charges (so $\rho = 0$) and no electric currents (so $\vec{J} = 0$). Then Maxwell's eq'ns become

$$\nabla \cdot \vec{E} = 0 \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

where μ_0 = permeability of free space, ε_0 = permittivity of free space; constants (property of vacuum (aka "free space"))

Then we have:

$$\nabla \times (\nabla \times \vec{E}) = \nabla(\cancel{\nabla \cdot \vec{E}}) - \nabla^2 \vec{E}$$

0

$$\Rightarrow \nabla \times \underbrace{(\nabla \times \vec{E})}_{= -\frac{\partial \vec{B}}{\partial t}} = -\nabla^2 \vec{E}$$

(from Maxwell's eq'n written above)

$$\Rightarrow -\nabla \times \frac{\partial \vec{B}}{\partial t} = -\nabla^2 \vec{E}$$

⇒ can interchange $\nabla \times$ with $\frac{\partial}{\partial t}$

$$\Rightarrow -\frac{\partial}{\partial t} \underbrace{(\nabla \times \vec{B})}_{= \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}} = -\nabla^2 \vec{E}$$

(from above maxwell's eq'ns)

$$\Rightarrow \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \nabla^2 \vec{E}$$

So we have: $$\boxed{\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \nabla^2 \vec{E}}$$

← This is a wave eq'n in 3 Dimensions since by definition,

$$\nabla^2 \vec{E} = \nabla^2 (E_x, E_y, E_z) = \nabla^2 \underbrace{(E_x \hat{x} + E_y \hat{y} + E_z \hat{z})}_{= \vec{E}}$$

$$= (\nabla^2 E_x)\hat{x} + (\nabla^2 E_y)\hat{y} + (\nabla^2 E_z)\hat{z}$$

$$= \left(\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2}\right)\hat{x} + (\nabla^2 E_y)\hat{y} + (\nabla^2 E_z)\hat{z}.$$

Similarly,

$$\nabla\times(\nabla\times\vec{B}) = \nabla(\cancel{\nabla\cdot\vec{B}}) - \nabla^2\vec{B}$$

0 (By Maxwell's eq'n)

$$\Rightarrow \quad \nabla\times\underbrace{(\nabla\times\vec{B})}_{\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}} = -\nabla^2\vec{B} \quad \Rightarrow \quad \mu_0\varepsilon_0\nabla\times\left(\frac{\partial\vec{E}}{\partial t}\right) = -\nabla^2\vec{B}$$

$$\Rightarrow \quad \mu_0\varepsilon_0\frac{\partial}{\partial t}\underbrace{(\nabla\times\vec{E})}_{-\frac{\partial\vec{B}}{\partial t}} = -\nabla^2\vec{B}$$

$\leftarrow$ By Maxwell's eq'n

$$\Rightarrow \quad \mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2} = \nabla^2\vec{B}.$$

$$\Rightarrow \boxed{\frac{1}{\mu_0\varepsilon_0}\nabla^2\vec{B} = \frac{\partial^2\vec{B}}{\partial t^2}}$$

(Just like $\vec{E}$.)

* Wave eq'n in 3D.

Recall that by definition: $\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ ← Laplacian operator

And $\vec{B}(x,y,z) = B_x(x,y,z)\,\hat{x} + B_y(x,y,z)\,\hat{y} + B_z(x,y,z)\,\hat{z}$

$$\Rightarrow \nabla^2\vec{B} = (\nabla^2 B_x)\hat{x} + (\nabla^2 B_y)\hat{y} + (\nabla^2 B_z)\hat{z}$$

$$= \left(\frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2} + \frac{\partial^2 B_x}{\partial z^2}\right)\hat{x} + \left(\frac{\partial^2 B_y}{\partial x^2} + \frac{\partial^2 B_y}{\partial y^2} + \frac{\partial^2 B_y}{\partial z^2}\right)\hat{y}$$

$$+ \left(\frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial y^2} + \frac{\partial^2 B_z}{\partial z^2}\right)\hat{z}$$

And $$\frac{\partial^2\vec{B}}{\partial t^2} = \left(\frac{\partial^2 B_x}{\partial t^2}\right)\hat{x} + \left(\frac{\partial^2 B_y}{\partial t^2}\right)\hat{y} + \left(\frac{\partial^2 B_z}{\partial t^2}\right)\hat{z}$$

So matching $\hat{x}$ component of $\frac{\partial^2\vec{B}}{\partial t^2}$ with the $\hat{x}$ component of $\left(\frac{\nabla^2\vec{B}}{\mu_0\varepsilon_0}\right)$;

$\hat{y}$ " " " " " $\hat{y}$ " " " ;

and $\hat{z}$ " " " " " $\hat{z}$ " " " ;

we get

see next pg

We get the following:

$\hat{x}$ - component:

$$\frac{\partial^2 B_x}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \nabla^2 B_x$$

← one wave eq'n (in 3D.) (obeyed by the x-component of $\vec{B}$)

$\hat{y}$ - component:

$$\frac{\partial^2 B_y}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \nabla^2 B_y$$

← another wave eq'n (in 3D) (obeyed by the y-component of $\vec{B}$.)

$\hat{z}$ - component:

$$\frac{\partial^2 B_z}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \nabla^2 B_z$$

← another wave eq'n (in 3D). (obeyed by the z-component of $\vec{B}$.)

And $\boxed{c^2 = \frac{1}{\mu_0 \varepsilon_0}}$ $c \equiv$ speed of light in vacuum $= \boxed{3 \times 10^8 \text{ m/s.}}$

⊛ Note:

Don't worry about all the math here. I just want you to see where we get the wave eq'n for EM waves.

But I do expect you to know how to derive the ~~wave~~ EM wave eq'ns given ① relationship that $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$ (for any vector $\vec{A}$) and ② The 4 Maxwell' eq'ns.