MITES 2008 : Physics III : Oscillations & Waves

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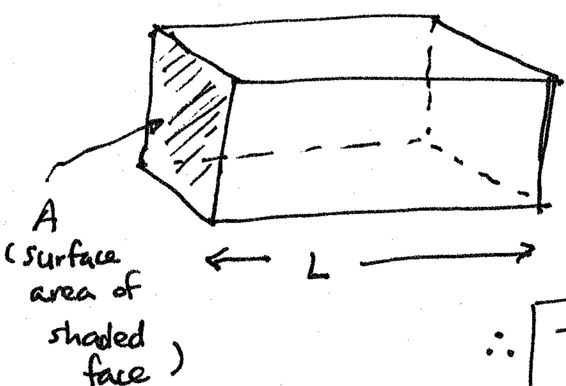
Solution set 0 :

Problem 1. Adding up infiniterimal quantities : Integration

(a) By the time the sticky sheet of paper reaches the end of corridor, it would have captured all the dust particles in its way (i.e. All the dust particles initially in the corridor would be stuck to the paper.). So we need to find the total # of dust particles in the corridor:



$$\rho = \frac{\text{total \# of particles}}{\text{unit volume.}}$$

↑ Constant throughout the corridor.

Volume of corridor = AL

$$\therefore \boxed{\text{Total \# particles in corridor} = \rho A L}$$

↑ This is the total # of dust particles collected by the sticky tape.

(b) Again, we need to find the total # of dust particles in the corridor.
But this time, $\rho(x) = kx$ ← not constant throughout the length of corridor.

First, notice that ρ has dimension of $\frac{1}{\text{Volume}} = \frac{1}{(\text{length})^3}$.
($\because \rho$ is # of particles (dimensionless) per unit volume.)

so $\rho(x) = kx$ (and x has dimension of Length)

$$\Rightarrow \frac{1}{(\text{length})^3} = k \cdot \text{length}$$

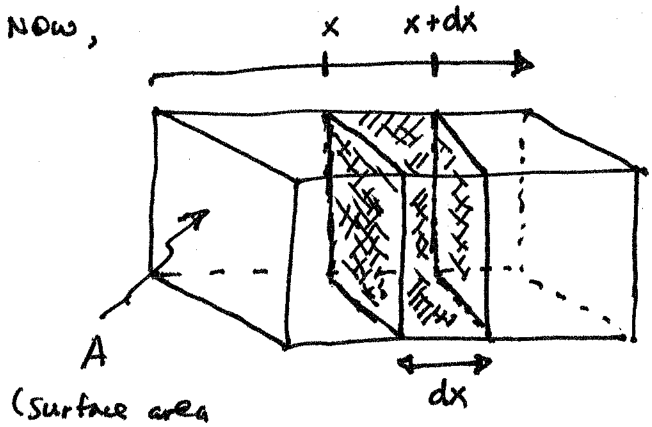
$$\Rightarrow \boxed{k \text{ has dimension of } \frac{1}{(\text{length})^4}}$$

(Also acceptable is $\frac{\text{\# particles}}{(\text{length})^4}$)

→ over

1(b) continued:

Now,



consider a rectangular slab of thickness dx as shown in the diagram to the left. This slab is located between the positions x and x+dx.

What is the total # of dust particles contained within this slab of infinitesimal thickness dx?

(i.e. dx is an extremely small thickness.)

To answer this, notice that at position x, the density is $\rho(x) = Kx$.

At x+dx, density is $\rho(x+dx) = K(x+dx)$.

But if dx is so small (infinitesimal), then the difference between $\rho(x)$ and $\rho(x+dx)$ is negligible; Hence we can ~~treat~~ treat the density within the infinitesimal slab to be uniform with the value $\rho(x) = Kx$.

And from (a), we know how to calculate the total # of particles contained in a box with uniform density; it's just:

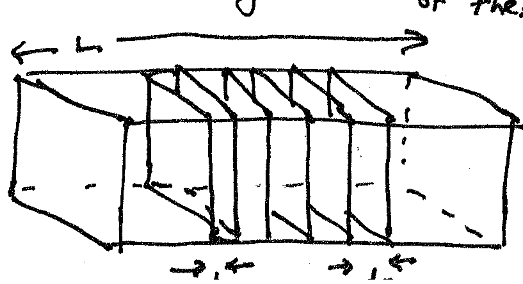
$$\rho(x) \cdot \text{Volume of slab} = \rho(x) \cdot (A\,dx)$$

Thickness of slab.

$$= \underbrace{(Kx\,A)dx}$$

total # of dust particles contained within the rectangular slab. (shaded portion in the above diagram.)

Now, the corridor is made up of many stacks of these ~~infinitesimal~~ rectangular slabs of infinitesimal thickness:



Hence,

Total # of dust Particles in the corridor = Sum of the total # of particles contained within each of the infinitesimal slabs

$$= \sum_{x=0}^{L} (k x A) dx$$

$$\text{"="} \int_{x=0}^{L} kxA\, dx$$

$$= \underbrace{kA}_{\text{constants}} \int_0^L x\, dx = \frac{kAx^2}{2}\Bigg|_0^L = \boxed{\frac{kAL^2}{2}}$$

∴ The total # of dust particles stuck to the face of sticky tape after traversing the entire length of corridor is $\dfrac{kAL^2}{2}$.

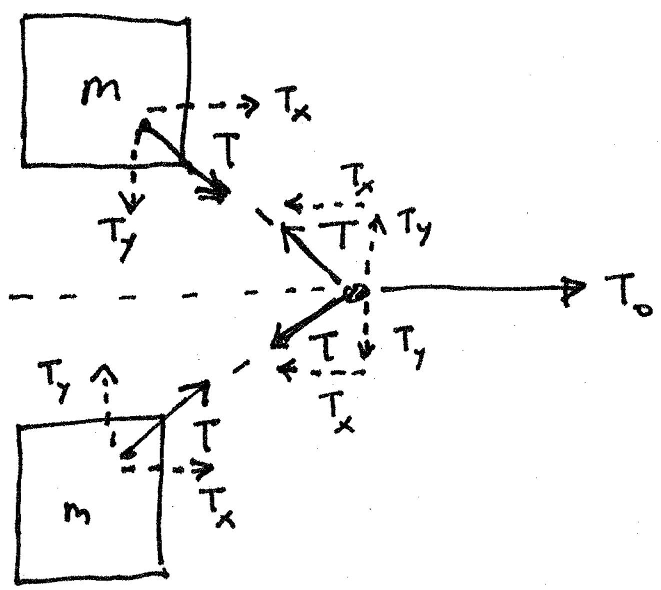
NOTE :

$$\sum_{x=a}^{b} f(x)\, dx \text{ "=" } \int_{x=a}^{b} f(x)\, dx$$

This is the "definition" of integral

(This is a key technique we'll be using throughout the course.)

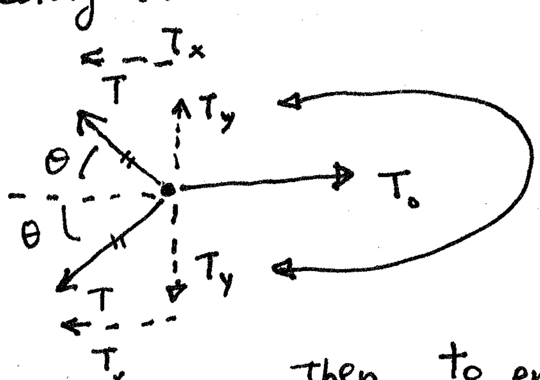
Problem 2 — Pulling two masses attached to a string



(a) Regardless of what θ is, the total force acting at the midpoint (or any other part of the massless string for that matter) must be zero.

Since, $F_{net} = \cancel{m}_{0} a = 0$. ($m=0$ $\because$ massless string).

(b) Focusing on the "dot" (midpt of string where you're exerting force T_0 w/ your hand):



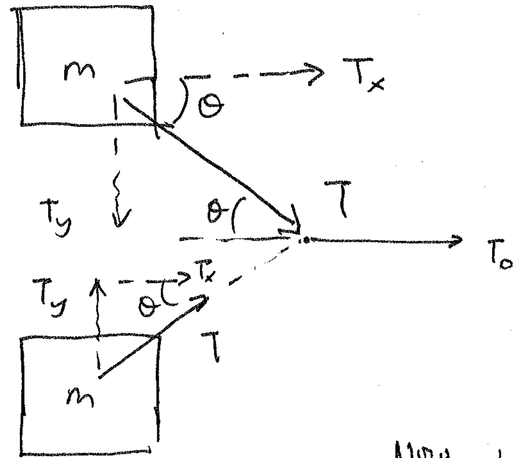
these 2 opposing vertical component forces cancel each other out.

Then, to ensure that $F_{net} = 0$ at the midpoint of string, we ~~only~~ need to make sure that the net horizontal force there is zero.

Looking at free body diagram: $2T_x = T_0$

But, $T_x = T\cos\theta \Rightarrow \boxed{T = \dfrac{T_0}{2\cos\theta}}$

(c)

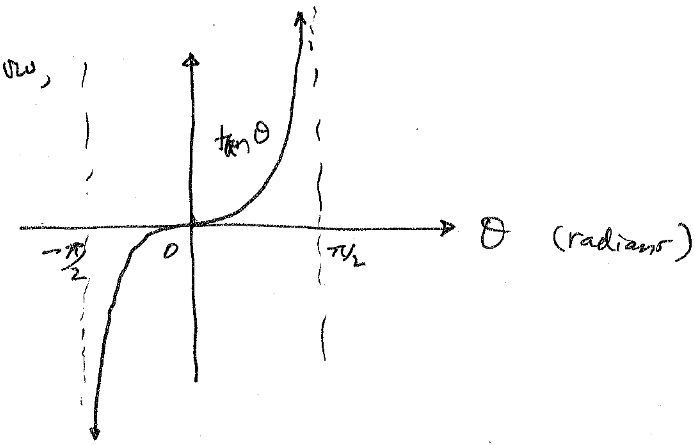


Vertical force acting on one block

$$T_y = T\sin\theta$$
$$= \boxed{\frac{T_0}{2}\tan\theta}$$

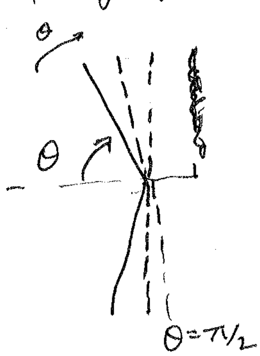
* For both upper & lower blocks.

Now,



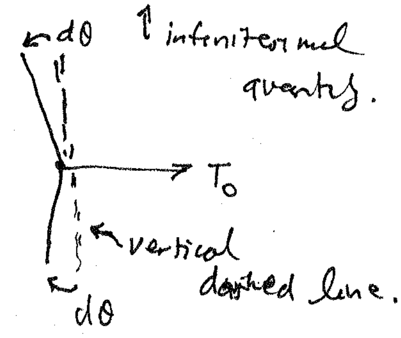
Hence, as $\theta \to \pi/2$, $T_y \to +\infty$. And the tension in the string would blow up to $+\infty$. (!)

~~The~~ ~~answer~~ This comes about because we need the F_{net} on the midpoint of string to be zero. But as $\theta \to \pi/2$, the string is approaching a perfectly vertical line. But since tension in a string, by definition, can only act parallel to the string itself, and the horizontal component of string disappears as $\theta \to \pi/2$, it is impossible for the string to exert a horizontal force when the string is perfectly vertical ($\theta = \pi/2$) and hence the requirement that there be no net force at the midpoint of string cannot be met when $\theta \to \pi/2$ (i.e. string cannot exert any horizontal force when it's perfectly vertical, so no force exists to cancel out the horizontal T_0 force exerted by your hand.).



Hence, the eq'n $T = \frac{T_0}{2\cos\theta}$ you derived in 2(b) is not applicable when $\theta = \pi/2$ (you'd be dividing by zero ($\cos(\pi/2) = 0$) there anyway; so can't be right!).

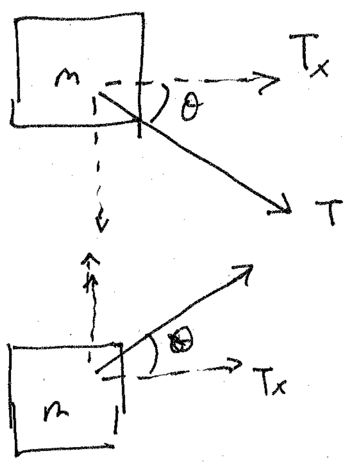
The assumption then, is that at $t=0$, θ is not exactly $\pi/2$ (Pg 6)
but slightly less than $\pi/2$ (i.e. $\theta(t=0) = \pi/2 - d\theta$)



so indeed, at $t=0$, the tension T in the string is very large (since $T = \frac{T_0}{2\cos\theta}$ and $\frac{1}{\cos\theta}$ is large when $\theta \sim \pi/2$)

but it's still finite (not infinite).

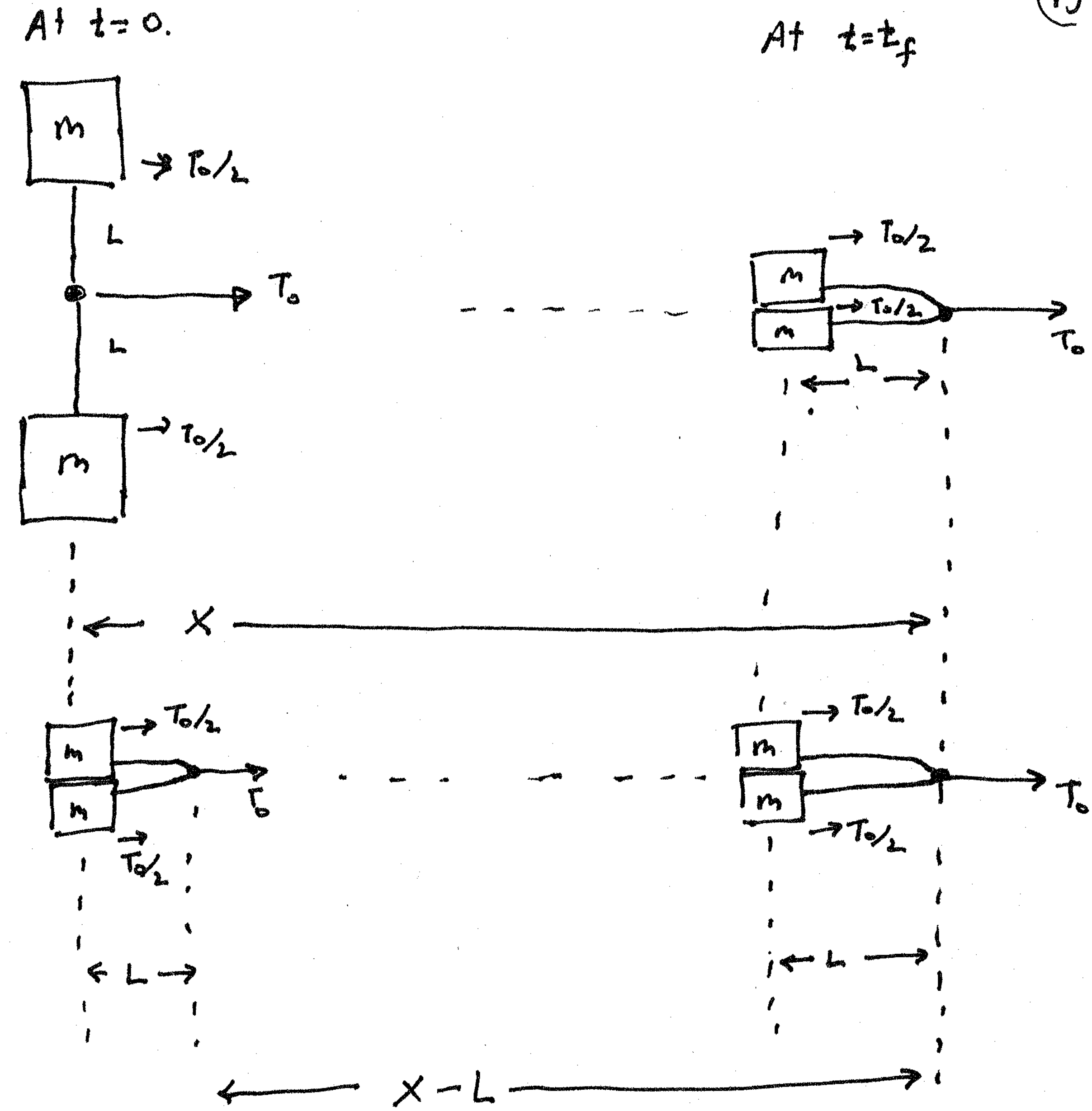
(d) From the free body diagram:



$$T_x = T\cos\theta$$
$$= \frac{T_0}{2\cancel{\cos\theta}}\cancel{\cos\theta}$$
$$= \boxed{\frac{T_0}{2}}$$

(e) & (f) (Now combined into one problem)

Next pg.



Top : scenario 1 , Bottom : scenario 2 .

- At $t=0$, the 2 blocks in scenario 1 are lined up with the position of the 2 blocks in scenario 2. All 4 blocks are initially at rest. The only difference between the 2 scenarios is that in scenario 2, the 2 blocks are starting out already glued to each other where as in scenario 1, 2 blocks are initially apart vertically. In both scenarios, the horizontal force on each block is $T_0/2$. And in both scenarios, the horizontal force on each block continues to be $T_0/2$ regardless of at what point in transit they are at.

- Since all 4 blocks are initially all lined up at $t=0$, all have horizontal component force $T_0/2$ throughout the motion, and all of them are starting w/ same initial velocity ($=0$), they must all have the same horizontal acceleration of $\frac{T_0}{2m}$. Therefore, by the time the 2 blocks in scenario 1 come together (at time $t=t_f$), the ~~blocks~~ horizontal (~~...~~) portion of 2 blocks in scenario 2 must still be lined up with the horizontal portion of the 2 blocks in scenario 1 at $t=t_f$. (as shown in the diagram).

And ~~...~~, right after $t=t_f$, all 4 blocks have the same horizontal velocity, hence all 4 blocks must have the same kinetic energy after $t=t_f$.

Now, the only source of energy input is the work that your hand does.

- In scenario 1, your hand moved distance of X.
- In scenario 2, your hand moved distance of $X-L$.

$\therefore$ In scenario 1: $W_{Hand} = T_0 X$

In scenario 2: $W_{Hand} = T_0 (X-L)$.

work done by your hand in 2 scenarios.

- In scenario 2, no loss of energy ever occurs. Hence, just after $t=t_f$,
- the total KE of 2 blocks is $T_0(X-L)$.

Now, as mentioned above, the total KE of 2 blocks in scenario 1 just after $t=t_f$ (just after sticking) must be the same as in scenario 2. But your hand did more work in scenario 1 ($T_0 X - T_0(X-L) = T_0 L$ more work done up to $t=t_f$).

Hence, that extra energy $T_0 L$ must be the energy that's lost due to the sticking of 2 blocks in scenario 1.

- In this inelastic collision, the energy is lost as heat (either to surrounding air / floor, or to increase the thermal jiggling of atoms making up the 2 blocks)
  (In that case, the blocks must be hotter after collision than before.)

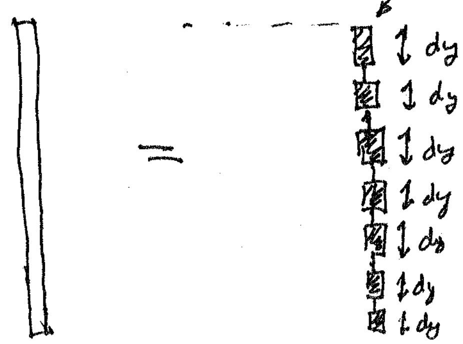
- The total energy of the universe is still conserved, but now distributed to different forms of energy after the collision.

$\boxed{\quad}$

Problem 3 : Falling rope

(a) $\boxed{\lambda = m/L}$ ← linear mass density

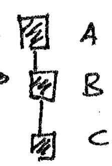
(b) **(i)**



each infinitesimal "block" of length dy has infinitesimal mass dm:

$$\boxed{\begin{aligned} dm &= \lambda(dy) \\ &= \frac{m}{L}(dy) \end{aligned}}$$

(ii) Since each infinitesimal block is in free fall, each infinitesimal segment block feels no tension. ~~Hence~~,
(when you're in free fall, you don't feel any force because nothing else is resisting your fall (e.g. if this block B →



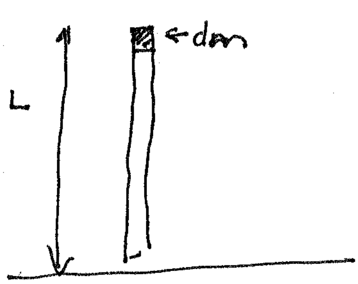
resisted fall of block A, then B would exert force on A (and A would exert force on B) That would be tension.)

Hence, only gravity is acting on each dm block.

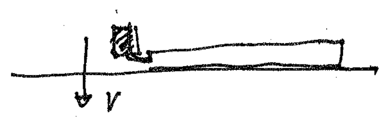
Thus:
the work done by gravity in moving the infinitesimal block at the top of rope to the ground is:

$$(dm)gL = \frac{1}{2}(dm)v^2$$

v = speed of (dm) just before hitting ground



later time →



$$\Downarrow$$

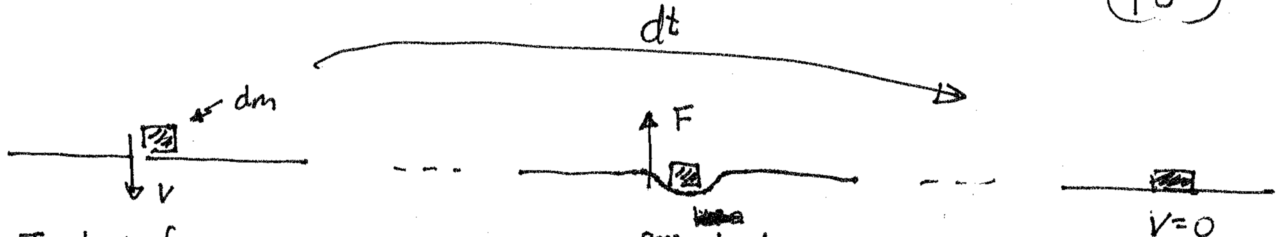
$$2gL = v^2$$

$$\Downarrow$$

$$\boxed{v = \sqrt{2gL}}$$

☒

(iii)



Just before hitting the ground.

ground depresses little while exerting ~~upforce~~ upward force F for infinitesimal time interval dt to stop the block

$\therefore$ Impulse = change in momentum

$\Rightarrow \quad F\,dt = (dm)V$

$\Rightarrow \quad F = \dfrac{(dm)}{dt} V$ $\qquad dm = \lambda\, dy$

$= \lambda \underbrace{\left(\dfrac{dy}{dt}\right)}_{=V} V$

$= \lambda V^2$ $\qquad V \equiv$ speed just before hitting ground.

$= \lambda \left(\sqrt{2gL}\right)^2$ $\qquad \lambda = m/L$

$= \boxed{2mg}$

$\Rightarrow$ Force exerted by the ground on (dm) to stop its motion is 2 mg.

(iv) The total force (weight) felt by the ground just after block (dm) shown above hits the ground is

F_{total} = force ground exerts on (dm) to stop it + force ground exerts to support the rest of rope that's already resting on the ground

$= 2mg + (\lambda L) g$

$= \boxed{3mg}$.