MITES 2008 : Physics III : Solution Set 5

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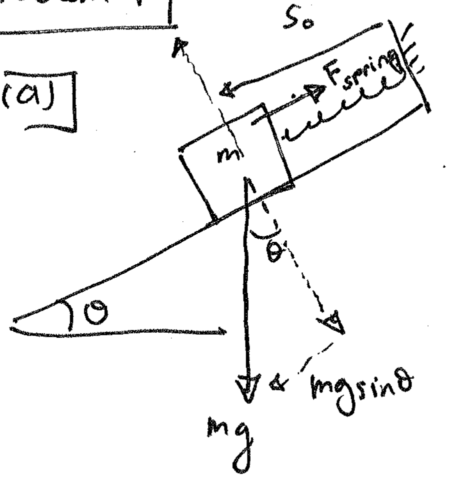
Problem 1

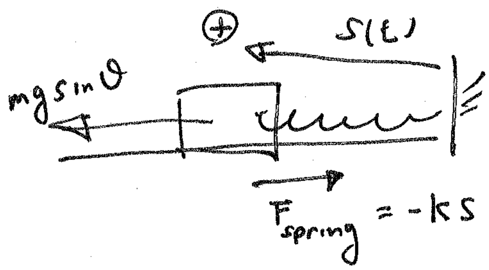
1(a)



In equilibrium:

$$+kS_o = mg\sin\theta \Rightarrow \boxed{S_o = \frac{mg}{k}\sin\theta}$$

1(b) Only concerned with forces parallel to the incline:



$$m\frac{d^2s}{dt^2} = mg\sin\theta + F_{spring}$$
$$= mg\sin\theta - ks$$
$$= kS_o - ks \quad \leftarrow \text{from (a)}$$
$$= -k(s - S_o)$$

$$\Rightarrow \frac{d^2s}{dt^2} + \omega_o^2(s - s_o) = 0 \quad \text{where} \quad \boxed{\omega_o = \sqrt{\frac{k}{m}}}$$

↑ Natural angular frequency.

But S_o constant $\Rightarrow \frac{d^2}{dt^2}(s - S_o) = \frac{d^2 s}{dt^2}$

∴ Letting $\tilde{S}(t) \equiv S(t) - S_o$ ← Displacement from equilibrium position S_o,

EOM becomes:

$$\boxed{\frac{d^2\tilde{s}}{dt^2} + \omega_o^2\tilde{s} = 0} \quad \leftarrow \text{SHM.}$$

(c) General solution: $\boxed{\tilde{s}(t) = C\cos(\omega_0 t - \phi)}$

2 free parameters; $C \equiv$ Amplitude $\qquad \phi \equiv$ phase shift.

How much you pull the block initially. $\qquad$ when you "start" the timer.

(d) $E_{total} = KE + PE = \frac{mV^2}{2} + \frac{k\tilde{s}^2}{2}$

$V = \dot{\tilde{s}}$

$= \boxed{\frac{m\dot{\tilde{s}}^2}{2} + \frac{k\tilde{s}^2}{2}}$ ← Notice that we've set the zero of the spring potential energy to be at when the mass is in equilibrium. Otherwise, we need to include the gravitational potential energy

(i.e. $E_{tot} \neq \frac{m\dot{s}^2}{2} + \frac{ks^2}{2}$.

It must be $E_{tot} = \frac{m\dot{\tilde{s}}^2}{2} + \frac{k\tilde{s}^2}{2}$.)

(e) $\frac{dE_{total}}{dt} = 0$ ← Conservation of energy

$$\frac{dE_{total}}{dt} = 0 = \frac{m}{2}\frac{d}{dt}\dot{\tilde{s}}^2 + \frac{k}{2}\frac{d}{dt}(\tilde{s}^2)$$

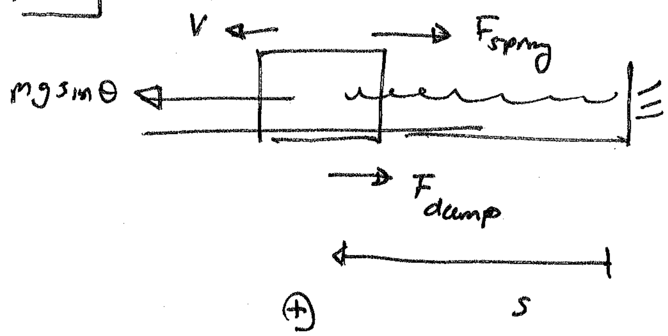
$$= \frac{m}{2}2\dot{\tilde{s}}\ddot{\tilde{s}} + \frac{k}{2}2\tilde{s}\dot{\tilde{s}}$$

$$= \dot{\tilde{s}}\left[m\ddot{\tilde{s}} + k\tilde{s}\right]$$

But $\dot{\tilde{s}}(t) \neq 0$ for many t.

$\Rightarrow \boxed{m\ddot{\tilde{s}} + k\tilde{s} = 0} \rightarrow$ EOM derived again.

(f.) Along the incline, the forces are now:



$$m\ddot{s} = -ks + mg\sin\theta - b\dot{s}$$
$$= -ks + ks_0 - b\dot{s}$$
$$= -k(s-s_0) - b\underbrace{\frac{d}{dt}(s-s_0)}_{=\frac{ds}{dt}\ \because\ s_0 \text{ is constant}}$$

$$\Rightarrow \underbrace{m\ddot{s}}_{=m\ddot{\tilde{s}}} = -k\tilde{s} - b\dot{\tilde{s}}$$

$$\Rightarrow \ddot{\tilde{s}} + \frac{b}{m}\dot{\tilde{s}} + \omega_0^2\tilde{s} = 0$$

Let $2\gamma \equiv b/m$ so we have;

$$\boxed{\ddot{\tilde{s}} + 2\gamma\dot{\tilde{s}} + \omega_0^2\tilde{s} = 0} \quad \leftarrow \text{EOM}$$

(g.) Guess solution to be: $\tilde{s}(t) = Ae^{i\alpha t}$ ← for now, a $\mathbb{C}$ number. I'll take the real solution at the end.

plugging into EOM we get:

$$-\alpha^2 + 2\gamma i\alpha + \omega_0^2 = 0.$$

Solving this quadratic eqn yields: $\alpha_\pm = \dfrac{-2\gamma i \pm \sqrt{-4\gamma^2 + 4\omega_0^2}}{-2}$

$$\Rightarrow \alpha_\pm = i\gamma \pm \sqrt{\omega_0^2 - \gamma^2}$$

∴ 2 solutions (one for α_+, the other for α_-). By linearity, sum of these 2 solutions is also a solution. In fact, it's the most general solution:

$$\Rightarrow \tilde{s}(t) = Ae^{i\alpha_+ t} + Be^{+i\alpha_- t}$$
$$= \boxed{e^{-\gamma t}\left[Ae^{i\tilde{\omega}t} + Be^{-i\tilde{\omega}t}\right]} \quad \text{where } \tilde{\omega} \equiv \sqrt{\omega_0^2 - \gamma^2}.$$

Note: For solution to damped OHM, leaving your answer in $\mathbb{C}$ # form is fine. (because it's complicated.).

When the oscillator is critically damped, it comes to rest most rapidly without oscillating. This occurs when $\tilde{\omega} = \sqrt{\omega_0^2 - \gamma^2} = 0$

$$\Rightarrow \boxed{\omega_0 = \gamma}.$$

$\boxed{(h)}$ $\tilde{s}(t) = e^{-\gamma t}\left[Ae^{i\tilde{\omega}t} + Be^{-i\tilde{\omega}t}\right]$

when underdamped, $\tilde{\omega}$ is real #. so, we can extract the real solution from this $Ae^{i\tilde{\omega}t} + Be^{-i\tilde{\omega}t}$ to be $C\cos(\tilde{\omega}t - \phi)$.

Real solution is: $\tilde{s}(t) = e^{-\gamma t} C\cos(\tilde{\omega}t - \phi)$.

$$(E_{total}(t)) = \frac{k}{2}(\text{Amplitude at time } t)^2$$

$$= \frac{k}{2} C^2 e^{-\gamma t 2}.$$

$\therefore$ E_{total} decays due to $e^{-2\gamma t}$ factor. ($\because$ Amplitude $Ce^{-\gamma t}$ is decaying.)

$\therefore$ After $\Delta t = \frac{2}{\gamma}$:

$$\Delta E_{tot} = E_{tot}(t = \tfrac{2}{\gamma}) - E_{tot}(t=0)$$

$$= \frac{kC^2}{2}\left[e^{-4} - 1\right] < 0 \Rightarrow \Delta E_{tot} < 0.$$

$$\Rightarrow \boxed{|\Delta E_{tot}| = \frac{kC^2}{2}\left[1 - e^{-4}\right]}$$

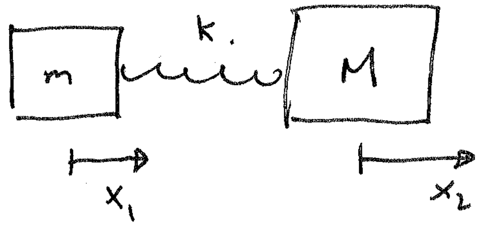
← maximal heat energy that can be delivered to the ramp after $\Delta t = 2/\gamma$

1(i) ω_0 doesn't change when $g \to 2g$.

Nothing changes when $g \to 2g$ except for the value of S_0.

So now changes to $S_0 = \frac{m2g}{k} \sin\theta$.

Problem 2 Coupled Oscillators:



EoMs:

$$\left.\begin{aligned} m\ddot{x}_1 &= +k(x_2-x_1) \\ M\ddot{x}_2 &= -k(x_2-x_1) \end{aligned}\right\} \Rightarrow \begin{aligned} \ddot{x}_1 &= \frac{k}{m}(x_2-x_1) \\ \ddot{x}_2 &= \frac{-k}{M}(x_2-x_1) \end{aligned}$$

writing in matrix form:

$$\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} = \begin{pmatrix} \frac{-k}{m} & \frac{k}{m} \\ \frac{k}{M} & \frac{-k}{M} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

To solve: guess $\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{pmatrix} A \\ B \end{pmatrix} e^{i\alpha t}$

α is yet to be specified. (we'll determine it after plugging in.)

Plugging into the matrix EOM:

$$-\alpha^2 \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} -\frac{k}{m} & \frac{k}{m} \\ \frac{k}{M} & \frac{-k}{M} \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix}$$

$$= \begin{pmatrix} -\alpha^2 & 0 \\ 0 & -\alpha^2 \end{pmatrix}$$

$$\Rightarrow \quad 0 = \left\{ \begin{pmatrix} \alpha^2 & 0 \\ 0 & \alpha^2 \end{pmatrix} + \begin{pmatrix} -\frac{k}{m} & \frac{k}{m} \\ \frac{k}{M} & -\frac{k}{M} \end{pmatrix} \right\} \begin{pmatrix} A \\ B \end{pmatrix}$$

To simplify notation, let $\omega_1^2 = \frac{k}{m}$ $\quad \omega_2^2 = \frac{k}{M}$.

$$\Rightarrow \quad 0 = \underbrace{\begin{bmatrix} \alpha^2 - \omega_1^2 & \omega_1^2 \\ \omega_2^2 & \alpha^2 - \omega_2^2 \end{bmatrix}}_{\text{"} \; C} \begin{bmatrix} A \\ B \end{bmatrix}$$

Need $\det(C) = 0$ for non-trivial solution. (i.e. $\begin{pmatrix} A \\ B \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$)

$$\Rightarrow \quad 0 = \det(C) = (\alpha^2 - \omega_1^2)(\alpha^2 - \omega_2^2) - \omega_1^2 \omega_2^2$$

$$= \alpha^4 - (\omega_1^2 + \omega_2^2)\alpha^2$$

$$\Rightarrow \quad 0 = \alpha^2 \left[\alpha^2 - (\omega_1^2 + \omega_2^2) \right]$$

$$\Rightarrow \quad \alpha_1 = 0 \quad \text{or} \quad \alpha_2 = \pm\sqrt{\omega_1^2 + \omega_2^2}$$

2 solutions. $\Rightarrow$ Normal mode angular frequencies are:

$$\boxed{\begin{aligned} \alpha_1 &= 0 \\ \alpha_2 &= \sqrt{\omega_1^2 + \omega_2^2} \end{aligned}}$$

First, for $\underline{\alpha_2 = \pm\sqrt{\omega_1^2 + \omega_2^2}}$;

From our guess: $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} A \\ B \end{pmatrix} e^{i\alpha_{2,2} t}$ Need to determine A & B.

From the matrix Eqn on pg 6, we have:

$$0 = \begin{bmatrix} \alpha_2^2 - \omega_1^2 & \omega_1^2 \\ \omega_2^2 & \alpha_2^2 - \omega_2^2 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

$$= \begin{bmatrix} \omega_2^2 & \omega_1^2 \\ \omega_2^2 & \omega_1^2 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

$$\Rightarrow \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \omega_2^2 A + \omega_1^2 B \\ \omega_2^2 A + \omega_1^2 B \end{pmatrix} \Rightarrow \omega_2^2 A = -\omega_1^2 B$$

$$\Rightarrow \boxed{A = -\left(\frac{\omega_1}{\omega_2}\right)^2 B}$$

$$\Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = A_1 \begin{pmatrix} 1 \\ -\left(\frac{\omega_2}{\omega_1}\right)^2 \end{pmatrix} e^{i\alpha_2 t} + A_2 \begin{pmatrix} 1 \\ -\left(\frac{\omega_2}{\omega_1}\right)^2 \end{pmatrix} e^{-i\alpha_2 t}$$

$\downarrow$ Real, physically meaningful version is: $\alpha_2 = \sqrt{\omega_1^2 + \omega_2^2}$

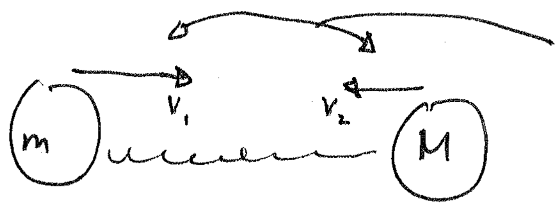
$$\boxed{\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = C \begin{pmatrix} 1 \\ -\left(\frac{\omega_2}{\omega_1}\right)^2 \end{pmatrix} \cos(\alpha_2 t - \phi_2)}$$

... One of 2 normal modes.

And the normal coordinate corresponding to mode α_2 is:

$$\boxed{\begin{pmatrix}1\\ -\left(\frac{\omega_2}{\omega_1}\right)^2\end{pmatrix}} = \begin{pmatrix}x_1\\ x_2\end{pmatrix}$$

This normal coordinate describes anti-symmetric motion where:



these two speeds are the same if and only if $\omega_1 = \omega_2$. (i.e. $m = M$)
(since we'd have $\begin{pmatrix}x_1\\ x_2\end{pmatrix} = \begin{pmatrix}1\\ -1\end{pmatrix}$.)

Notice that if $\omega_2 < \omega_1$; then $\left(\frac{\omega_2}{\omega_1}\right)^2 < 1$.
(i.e. $M > m$)

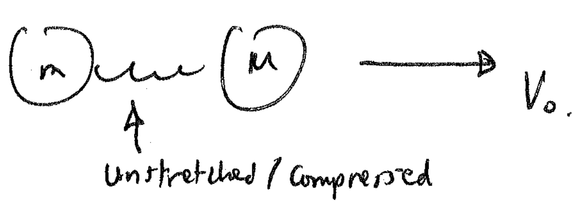
This means that x_2 (mass M) oscillates with less (smaller) amplitude compared to amplitude of m.

Inversely, if $m > M$, then $\omega_2 > \omega_1 \Rightarrow \left(\frac{\omega_2}{\omega_1}\right)^2 > 1$.

$\Rightarrow$ Amplitude of M is larger than that of m.

· As for $\alpha_1 = 0$ mode:

$$\boxed{\begin{pmatrix}x_1\\ x_2\end{pmatrix} = V_0 t\begin{pmatrix}1\\ 1\end{pmatrix} + \begin{pmatrix}1\\ 1\end{pmatrix}C_1.}$$



Pure translational motion.

Whether $m > M$, or $m = M$, or $m < M$, nothing about this normal mode changes.

∴ The most general solution is:

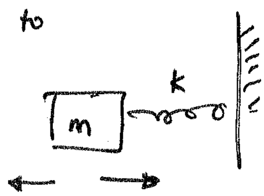
$$\boxed{\begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix} = (V_0 t + C_1)\begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ -\left(\frac{\omega_2}{\omega_1}\right)^2 \end{pmatrix} \cos\left(\sqrt{\omega_1^2+\omega_2^2}\, t - \varphi_2\right)}$$

Limiting case:

If $m << M$ then $\omega_2 << \omega_1 \Rightarrow \left(\frac{\omega_2}{\omega_1}\right) \simeq 0$.

This means M hardly oscillates at all.

This correspond to (As if $M \to \infty$).



Hard wall (like M)

$\omega_1 = \sqrt{k/m}$

Problem 3: See solution set 3 First problem there.

Problem 4 (a) Let $g(z)$ be any one-variable (z) function.

Then by plugging in $z = x \pm Vt$;

Check:

$$\frac{\partial^2 g(x \pm Vt)}{\partial t^2} = \frac{\partial}{\partial t}\left(\frac{\partial g}{\partial t}\right) = \frac{\partial}{\partial t}\left(\frac{dg}{dz}\cdot \underbrace{\frac{\partial z}{\partial t}}_{\pm V}\right) = \pm V \frac{\partial}{\partial t}\left(\frac{dg(z)}{dz}\right)$$

$$= (\pm V)\left[\frac{d^2 g(z)}{dz^2}\cdot \underbrace{\frac{\partial z}{\partial t}}_{(\pm V)}\right] = V^2 \frac{d^2 g(z)}{dz^2}$$

Pg 10

And, $v^2 \frac{\partial^2 g(x \pm vt)}{\partial x^2}$

$= v^2 \frac{\partial}{\partial x}\left[\frac{\partial g}{\partial x}\right]$

$= v^2 \frac{\partial}{\partial x}\left[\frac{dg}{dz} \cdot \underbrace{\frac{\partial z}{\partial x}}_{=1}\right]$

$= v^2 \frac{\partial}{\partial x}\left(\frac{dg}{dz}\right)$

$= v^2 \left(\frac{d^2 g}{dz^2}\right) \cdot \underbrace{\frac{\partial z}{\partial x}}_{=1}$

$= v^2 \frac{d^2 g}{dz^2}$

Hence:

$$\frac{\partial^2 g(x \pm vt)}{\partial t^2} = v^2 \frac{\partial^2 g(x \pm vt)}{\partial x^2}$$

$\Downarrow$

~~Hence~~ $f_+(x,t) = g(x+vt)$ is a sol'n.

as well as

$f_-(x,t) = g(x-vt)$.

And by linearity, $\boxed{\begin{aligned} f(x,t) &= f_+(x,t) + f_-(x,t) \\ &= g(x+vt) + g(x-vt) \end{aligned}}$ ← general solution to wave eq'n.

1(b)

$\mathbb{C}$-number version of plane wave moving to the right with speed v is:

$\tilde{g}_+(x,t) = \boxed{\tilde{A} e^{i(kx - \omega t)}}$ where $\omega = kv$.

$= \tilde{A} e^{ik(x - vt)}$ and $k = 2\pi/\lambda$.

~~Thus~~

↑ We can immediately tell that this is a solution to the $\mathbb{C}$-# version of the wave eq'n since it has the form: $g(z) = \tilde{A} e^{ikz}$ → $g(x-vt) = \tilde{A} e^{ik(x-vt)}$

Thus indeed; $\frac{\partial^2 \tilde{g}_+(x,t)}{\partial t^2} = v^2 \frac{\partial^2 \tilde{g}_+(x,t)}{\partial x^2}$. ☒

1(c) Sinusoidal plane wave, moving to the right with speed v and wavelength λ is:

$$\boxed{f_+(x,t) = A\sin(kx - \omega t) \quad \text{where} \quad k = 2\pi/\lambda, \ \omega = kv}$$

Again, we know immediately that $f_+(x,t)$ must satisfy the wave eq'n since it has the form:

$$f_+(x,t) = g(x - vt)$$

where $g(z) = A\sin(kz)$.

1(d) Incident wave: $f_i(x,t) = A_i e^{i(k_1 x - \omega t)}$ $(x \leq 0)$

$k_1 = \frac{2\pi}{\lambda_1}$

reflected wave

$f_r(x,t) = A_r e^{i(-k_1 x - \omega t)}$ $(x \leq 0)$

$\lambda_1 \equiv$ wavelength in string A.

Transmitted wave $f_t(x,t) = A_t e^{i(k_2 x - \omega t)}$ $(x \geq 0)$ $\lambda_2 \equiv$ wavelength in string B.

1(e) · Resultant wave in string A: $\tilde{y}_1(x,t) = f_i + f_r$

$$= \left[A_i e^{ik_1 x} + A_r e^{-ik_1 x}\right] e^{-i\omega t}$$

· Resultant wave in string B: $\tilde{y}_2(x,t) = f_t(x,t)$

$$= A_t e^{i(k_2 x - \omega t)}$$

[f] 2 boundary conditions (BC) at $x=0$:

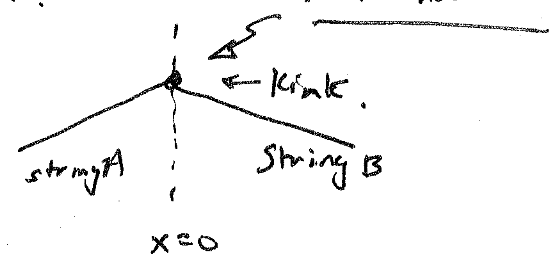
BC 1: 2 strings are joined to each other at $x=0$:

$$\Rightarrow \quad \tilde{y}_1(x=0,t) = \tilde{y}_2(x=0,t)$$

$$\Rightarrow \quad \boxed{A_i + A_r = A_t}$$

BC2: No kinks at the joint:

i.e.



$$\Rightarrow \quad \left.\frac{\partial \tilde{y}_1}{\partial x}\right|_{x=0} = \left.\frac{\partial \tilde{y}_2}{\partial x}\right|_{x=0}$$

$$\Rightarrow \quad ik_1(A_i - A_r) = ik_2 A_t$$

$$\Rightarrow \quad \boxed{k_1(A_i - A_r) = k_2 A_t}$$

Problem 5 Fourier Series

1(a) Done in class notes. ✓

Ans: Each normal mode is described by a positive integer n. $(n=1,2,3,\ldots)$

$$k_n = \frac{n\pi}{L} \Rightarrow \omega_n = k_n V = \frac{n\pi V}{L}$$

$$\frac{2\pi}{\lambda_n} = k_n$$

$$\Rightarrow \boxed{\lambda_n = \frac{2L}{n}}$$

And normal mode "n" is: $y_n(x,t) = \left[A_n \sin\left(\frac{n\pi v t}{L}\right) + B_n \cos\left(\frac{n\pi v t}{L}\right)\right] \sin\left(\frac{n\pi x}{L}\right)$

BCs are: $y(x=0,t)=0$

$y(x=L,t)=0$.

1(b) General sol'n:

$$\boxed{\begin{aligned} y(x,t) &= \sum_{n=1}^{\infty} y_n(x,t) \\ &= \sum_{n=1}^{\infty} \left[A_n \sin\left(\frac{n\pi v t}{L}\right) + B_n \cos\left(\frac{n\pi v t}{L}\right)\right] \sin\left(\frac{n\pi x}{L}\right) \end{aligned}}$$

1(c) Initial Conditions:

① $g(x) = y(x, t=0) = \frac{L}{4} \sin\left(\frac{8\pi x}{2L}\right) = \frac{L}{4} \sin\left(\frac{4\pi x}{L}\right)$.

② $\left.\frac{\partial y}{\partial t}\right|_{t=0} = 0.$ ← ∵ Initially, string isn't moving.

So: $g(x) = \frac{L}{4}\sin\left(4\frac{\pi x}{L}\right)$

$$= y(x, t=0)$$

$$= \sum_{n=1}^{\infty} y_n(x, t=0)$$

$$= \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{L}\right).$$

$$\Rightarrow B_m = \frac{2}{L}\int_0^L g(x)\sin\left(\frac{m\pi x}{L}\right)dx$$

$$= \frac{2}{L}\frac{L}{4}\underbrace{\int_0^L \sin\left(\frac{4\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right)dx}_{}$$

↑ zero unless $m=4$.

~~taken~~ $\Rightarrow \underline{B_m = 0 \ \left(\text{if } m \neq 4\right)}$

And when $m=4$: $B_4 = \frac{L}{4}\left(\underbrace{\frac{2}{L}\int_0^L \sin\left(\frac{4\pi x}{L}\right)\sin\left(\frac{4\pi x}{L}\right)dx}_{=1}\right)$

$$= \frac{L}{4}.$$

$$\therefore B_m = \begin{cases} L/4 & ; \text{if } m=4 \\ 0 & ; \text{if } m=0 \end{cases}$$

The 2nd initial condition gives:

$$0 = \left.\frac{\partial y}{\partial t}\right|_{t=0} = \sum_{n=1}^{\infty} \underbrace{(\omega_n A_n)}_{\substack{\shortparallel \\ C_n}} \sin\left(\frac{n\pi x}{L}\right)$$

C_n ← rename constant.

$$C_n = \frac{2}{L}\int_0^L 0 \cdot \sin\left(\frac{n\pi x}{L}\right) dx = 0 \quad \text{for } \underline{\text{all}}\ n.$$

$$\Rightarrow \omega_n A_n = C_n = 0 \quad \Rightarrow \quad \boxed{A_n = 0} \quad \text{for all } n.$$

Thus: Subsequent shape of string $(t > 0)$ is:

$$y(x,t) = \sum_{n=1}^{\infty} y_n(x,t)$$

$$= \sum_{n=1}^{\infty} B_n \cos\left(\frac{n\pi v t}{L}\right)\sin\left(\frac{n\pi x}{L}\right)$$

$$= B_4 \cos\left(\frac{4\pi v t}{L}\right)\sin\left(\frac{4\pi x}{L}\right)$$

$$= \boxed{\frac{L}{4} \cdot \cos\left(\frac{4\pi v t}{L}\right)\sin\left(\frac{4\pi x}{L}\right)}$$

Problem 6. EM waves

1(a)

$$\nabla \times (\underbrace{\nabla \times \vec{E}}) = \nabla(\cancel{\nabla \cdot \vec{E}}_{0}) - \nabla^2 \vec{E}$$

$$\Rightarrow \nabla \times \left(-\frac{\partial \vec{B}}{\partial t} \right) = -\nabla^2 \vec{E}$$

$$\Rightarrow -\frac{\partial}{\partial t} (\underbrace{\nabla \times \vec{B}}) = -\nabla^2 \vec{E}$$

$$\Rightarrow \frac{\partial}{\partial t} \left(\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right) = +\nabla^2 \vec{E}$$

$$\Rightarrow \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = +\nabla^2 \vec{E}$$

$$\Rightarrow \frac{\partial^2 \vec{E}}{\partial t^2} = \underbrace{\left(\frac{1}{\mu_0 \varepsilon_0} \right)}_{c^2} \nabla^2 \vec{E}$$

(c = speed of light in vacuum.)

$$\Rightarrow \boxed{\frac{\partial^2 \vec{E}}{\partial t^2} = c^2 \nabla^2 \vec{E}}$$

$$\nabla \times (\underbrace{\nabla \times \vec{B}}) = \nabla(\cancel{\nabla \cdot \vec{B}}_{0}) - \nabla^2 \vec{B}$$

$$\Rightarrow \nabla \times \left(\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right) = -\nabla^2 \vec{B}$$

$$\Rightarrow \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \underbrace{(\nabla \times \vec{E})}_{\left(-\frac{\partial \vec{B}}{\partial t}\right)} = -\nabla^2 \vec{B}$$

$$\Rightarrow \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} = \nabla^2 \vec{B}$$

$$\Rightarrow \frac{\partial^2 \vec{B}}{\partial t^2} = \underbrace{\left(\frac{1}{\mu_0 \varepsilon_0} \right)}_{c^2} \nabla^2 \vec{B}$$

$$\Downarrow$$

$$\boxed{\frac{\partial^2 \vec{B}}{\partial t^2} = c^2 \nabla^2 \vec{B}}$$

To show that $\vec{E} \perp \vec{B}$; we use these 2 maxwell's eqns

$$(\nabla \times \vec{E}) = -\frac{\partial \vec{B}}{\partial t}$$

and $$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}.$$

~~Plane wave EM~~

EM plane waves are described by:

$$\vec{E}(z,t) = \tilde{E}_0 \, e^{i(kz - \omega t)}$$

$$\vec{B}(z,t) = \tilde{B}_0 \, e^{i(kz - \omega t)}$$

where the <u>amplitude vectors are</u>:

$$\tilde{E}_0 = (E_{0x}, E_{0y}, \cancel{E_{0z}}^{\,0})$$

$$\tilde{B}_0 = (B_{0x}, B_{0y}, \cancel{B_{0z}}^{\,0})$$

Since EM wave is propagating in +Z direction and we showed in class that $\tilde{E}_0$ and $\tilde{B}_0$ thus <u>cannot</u> have a z-component.

Then:

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ yields

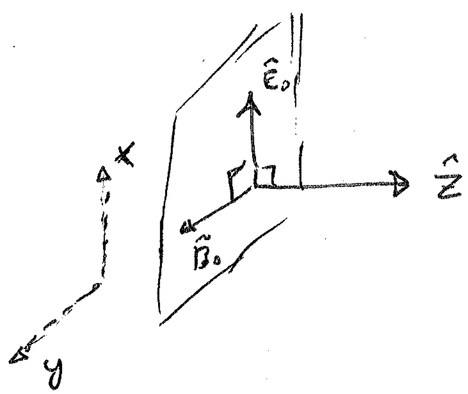
$$-k\,\tilde{E}_{0y} = \omega\,\tilde{B}_{0x}$$

and $$k\,\tilde{E}_{0x} = \omega\,\tilde{B}_{0y}.$$

Compactly written, these are:
$\Rightarrow$

$$\tilde{B}_0 = \frac{k}{\omega}(\hat{z} \times \tilde{E}_0)$$

$\Rightarrow$ $\tilde{B}_0$ and $\tilde{E}_0$ are perpendicular to each other



<u>Crecall</u>: cross product of 2 vectors (in this case, $\hat{z}$ & $\tilde{E}_0$) yields a 3rd vector that's perpendicular to both $\hat{z}$ & $\tilde{E}_0$.

From special relativity, recall that speed of light in vacuum is the same <u>pregardless</u> of the inertial reference frame you're in.

1(b) Planar EM wave moving in $+z$ direction

$$\begin{cases} \vec{E}(z,t) = \tilde{E}_0 \sin(kz - \omega t) \\ \vec{B}(z,t) = \tilde{B}_0 \sin(kz - \omega t) \end{cases} \qquad \omega = kc.$$

$\tilde{E}_0$ = amplitude of electric field component of EM wave.

$\tilde{B}_0$ = amplitude of magnetic field component of EM wave.

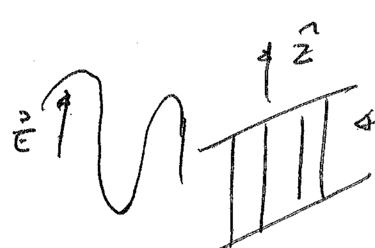
And from lecture: $\boxed{|\tilde{B}_0| = \dfrac{|\tilde{E}_0|}{c}}$

1(c) Since EM wave drives free e^-'s in conductors.

Hence energy from EM wave is transferred as work done on the ~~free~~ e^-'s in the conductor. $\Rightarrow$ EM wave dies away as it encounters a slab of conductor.

But insulator has no free e^-. So EM wave doesn't lose energy as it ~~passes~~ hits ~~insulator~~ a slab of insulator $\Rightarrow$ passes through freely.

1(d)



series of wires parallel to $+\hat{z}$ direction

Component of $\vec{E}$ vector that's parallel to the wire ($\hat{z}$-axis in this case) blocked out since it drives free e^- along the wire.

To block out natural light, use 2 wire-grid polarizers like this: