MITES 2009 : Calculus II - Final Examination

Massachusetts Institute of Technology
Instructor: Hyun Youk
TA: Nicholas Villalva
(Thursday, July 30, 2009 : 9:00 - 11:00 AM.)

Problem 1. Vectors:

(a.) {**2 points**}: Find a vector that is parallel to the line described by $(x, y, z) = (2, 1, -5) + t(16, 2, 9)$.

(b.) {**4 points**}: Find a vector that makes an angle of 60^o to $\hat{y}$ and makes 45^o angle with both $\hat{x}$ and $\hat{z}$.

Problem 2. Equation of a line: {**10 points**}:

Find the equation of the line passing through the point $(-2, \pi, 3)$ that intersects and is perpendicular to the line $(x, y, z) = (-1, -2, -1) + t(1, 2, 5)$.

Problem 3. Equation of a plane: {**6 points**}:

Consider the plane described by the equation $3x + 2y - \pi z - 2\pi = 0$. Find at least three points that lie on this plane. Write down two vectors that are normal to this plane.

Problem 4. Distance between a point and a line: {**10 points**}:

Show that the distance from the point (x_1, y_1) to the line $ax + by = c$ is $\frac{|ax_1 + by_1 - c|}{\sqrt{a^2 + b^2}}$.

Problem 5. Gradients and directional derivatives:

You are hiking on a mountain whose altitude is $z(x, y) = -x^3 + 3xy + y^2$.

(a.) {**3 points**}: Compute the gradient of z at position $(1, 2)$. (i.e., $\nabla z(1, 2)$).

(b.) {**6 points**}: Find the equation of a plane tangent to the graph of $z(x, y)$ at $(x, y) = (0, 2)$.

(c.) {**3 points**}: Write down any vector that is normal (i.e., perpendicular) to the plane you found in (b).

Problem 6. Ordinary Differential Equations: {**10 points**}:

Solve the following ordinary differential equation. Be sure to write down the most general solution of the equation which contains the right number of arbitrary constants. Here, y = y(x).

$2y''' + y'' - 5y' + 2y = 0$

Problem 7. Double and triple integrals: {**6 points**}:

$\int_{y=0}^{1} \int_{x=0}^{1} (xy)^4 cos(x^5) dx dy$

Problem 8. Flux and Divergences: {**10 points**}:

Consider a fluid whose velocity field is $\vec{v}(x, y, z) = (x - y, xyz, yz)$. Compute the flux of this fluid field through a rectangle described by the set of points $\{(x, y, z)|2 \leq x \leq 5, 2 \leq y \leq \pi, z = 1\}$.

Problem 9. Fourier Series:

Find the Fourier series representation for the following functions on the x-interval $[-\pi, \pi]$.

(a.) {**10 points**}: $f(x) = x$

(b.) {**6 points**}: $L(x) = cos(2x)cos(4x) - sin(2x)sin(4x) + cos(100x)sin(x) + sin(100x)cos(x)$.