MITES 2009 : Calculus II - Midterm Examination.

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(Wednesday, July 8, 2009 : 10:45 AM - 12:15 PM.)

Problem 1. Vectors {Total = 10 points}:

(a.) {**2 points**}: Find a vector that is parallel to the line described by $(x, y, z) = (1, -2, -2) + t(3, 16, -1)$.

(b.) {**4 points**}: Find a vector that is orthogonal (i.e., perpendicular) to the plane $x - 6y + z = 12$.

(c.) {**4 points**}: Find a vector that makes an angle of 30^o to $\hat{x}$ and makes equal angles with $\hat{y}$ and $\hat{z}$.

Problem 2. Equation of a line {Total = 20 points}:

(a.) {**5 points**}: Find the equation of the line that passes through $(-1, 2, -1)$ and is parallel to the $y - axis$.

(b.) {**5 points**}: Find the equation of the line passing through the two points $(0, 1, 1)$ and $(0, 1, 0)$.

(c.) {**10 points**}: Find the equation of the line passing through the point $(3, 1, -2)$ that intersects and is perpendicular to the line $(x, y, z) = (-1, -2, -1) + t(3, 3, 3)$.

Problem 3. Equation of a plane {Total = 20 points}:

(a.) {**6 points**}: Find the equation for the plane perpendicular to $(-1, 1, -1)$ and passing through $(1, 1, 1)$.

(b.) {**6 points**}: Find the equation for the plane perpendicular to the line described by $(x, y, z) = (1, 1, 1) + t(-2, 1, 2)$ and passing through $(-1, 1, 3)$.

(c.) {**8 points**}: Consider the plane described by the equation $Ax + Ay - Az - 17A = 0$. Show that this equation can be written in the equivalent form : $A(x - 3) + A(y - 5) - A(z + 9) = 0$. What can you then say about the point $(3, 5, -9)$? What can you then say about the vector $(A, A, -A)$?
(Hint: Think about how you solved Problem 1 (b)).

Problem 4. Integral of one variable function {Total = 10 points}:

(a.) {**5 points**}: $\displaystyle\int \frac{log(x)}{x} dx$ (Note: $log(x)$ is the natural log, base e.)
(Hint: Integrate using either the substitution or the "by parts" method).

(b.) {**5 points**}: $\displaystyle\int (log(x))^3 dx$
(Hint: Integrate by parts)

Problem 5. Orthogonal projection of a vector onto another vector {Total = 10 points}:

(a.) {**6 points**}: Find the orthogonal projection of the vector $(1, 1, 0)$ onto the vector $(1, -2, 0)$. What is the length of this projected vector?

(b.) {**4 points**}: Show that the distance from the point (x_1, y_1) to the line $ax + by = c$ is $\dfrac{|ax_1 + by_1 - c|}{\sqrt{a^2 + b^2}}$.

Problem 6. Hiking on a mountain $\{$Total = 20 points$\}$:

You are hiking on a mountain whose altitude is $z(x,y) = -x^2 - y^2 + 3$. Your position as a function of time t is $\vec{c}(t) = (1+4t, 2+7t)$. So $x(t) = 1+4t$ and $y(t) = 2+7t$ are the longitude and latitude of your position at time t respectively.

(a.) $\{$**2 points**$\}$: What is your altitude at time t?

(b.) $\{$**2 points**$\}$: Using your answer to (a), calculate the instantaneous rate of change in your altitude with respect to time, at $t = 3$.

Next, let's compute the same quantity as in (b) (i.e, $\frac{dz(\vec{c}(t))}{dt}$) but using directional derivatives computed along the trajectory $\vec{c}(t)$. To do this,

(c.) $\{$**3 points**$\}$: First, compute the gradient of z at position (x,y). (i.e., $\nabla z(x,y)$).

(d.) $\{$**3 points**$\}$: Next, compute your instantaneous velocity at time t on the trail. (i.e., $\frac{d\vec{c}(t)}{dt}$).

(e.) $\{$**5 points**$\}$: Finally, compute $\frac{dz(\vec{c}(t))}{dt}$ using the two quantities you calculated above.

(f.) $\{$**5 points**$\}$: Find the equation of a plane tangent to the graph of $z(x,y)$ at $(x,y) = (0,1)$.

Problem 7. Partial Differential Equation $\{$Total = 10 points$\}$:

Don't let the title of this problem scare you. All you need to do here is take some partial derivatives.

$f(t)$ is a function of one variable t. Let $u = g(x,y)$ where

$$g(x,y) = xyf(\frac{x+y}{xy}).$$

Show that u satisfies the following equation:

$$x^2\frac{\partial u}{\partial x} - y^2\frac{\partial u}{\partial y} = G(x,y)u$$

and find the function $G(x,y)$. Above equation is called a "partial differential equation"; we'll learn more about such equations later in the course.

Total Points for this exam : 100 Points.

But I will grade your exam out of 90 points.