MITES 2009 : Calculus II - Study Guide for Midterm Exam.

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(Midterm will be on Wed., July 8, 09: 10:45 AM - 12:15 PM.)

General Information about the Midterm exam: About 3/4 of the problems that will appear on the midterm will be very similar to the problems listed here or in the problem sets. So it is wise to review the solutions to the two problem sets and make sure that you understand how to derive the solutions yourself. The remaining 1/4 of the exam will require problem solving skills using the tools we developed in our class and on the problem sets. Use this handout to guide your preparation for the midterm exam. If you can do these problems, you are in a very good shape for the midterm exam.

1. **Hubble's law**. Hubble's law states that a galaxy recedes from another galaxy at velocity $\vec{v} = H\vec{r}$, where $\vec{r}$ is the position of the receding galaxy relative to the other galaxy. H is a scalar (number) called the "Hubble's constant" (independent of what galaxy you deal with). Suppose we represent a galaxy by a point. Then according to the Hubble's law, point A (one galaxy) would move away from point B (another galaxy) in a straight line joining the two points at velocity $\vec{v}$ defined above, where $\vec{r}$ is the vector emanating from point B and ending at point A. Now, suppose you have a third point (our planet Earth). And let $\vec{R}$ represent the position of galaxy B relative to the Earth. Applying the Hubble's law to Galaxy B as you watch it move away from the Earth, calculate the velocity with which the galaxy A moves away from the Earth.

2. Finding the equation of a line (e.g., Pset2, Probs. 3-5, 12).

3. Finding the equation of a plane (e.g., Pset2, Probs. 14-16, 18).

4. Finding the dot products between two vectors (e.g., Pset2, Probs. 9 & 10).

5. Finding and sketching level curves. (e.g., Pset2, Probs. 19-21).

6. Finding limits of multivariable functions (e.g. Pset2, Probs. 22-26).

7. Finding partial derivatives (e.g. Pset2, Probs. 27-29).

8. Finding equation of tangent planes (e.g., Pset 2, Probs. 30, 31).

9. Gradients and directional derivatives (e.g., Pset 2, Probs. 32, 33).
(Note: We will cover gradients and directional derivatives in more depth on Monday July 6th).

10. Sketching basic functions (e.g., Sin, Cos, 1/x, 1/(1+x^2), e^x, log(x), $x^2 - y^2 = 1$). (See Problem set 1).

11. Integrals from Problem set 1 (Probs. 11-14). Some of these will appear on the midterm exam. You'll need to show all your work in getting your answer, step-by-step.

12. Computing Taylor series. Also, reasoning through Taylor series: e.g., How do you approximate a function using a Taylor series?

13. Deriving and finding the length of a curve (See Lecture notes and Pset 1).

14. **Parametrized curves**. Sketch the following curves as a function of t. Be sure to indicate the direction of travel (as t increases) in your sketch. Also, compute the instantaneous "velocity" $\frac{d\vec{c}(t)}{dt}$. Notice that this is a vector.

(a.) $\vec{c}(t) = (cos(t), sin(t), \alpha t)$, α is a constant (often called the "pitch" of helicity).

(b.) $\vec{c}(t) = (vt - Rsin(\frac{vt}{R}), R - Rcos(\frac{vt}{R}))$ (R, v are constants, t is time). This parametrized curve is called the "cycloid". Consider a particle whose trajectory is described by $\vec{c}(t)$. By finding the instantaneous velocity $\frac{d\vec{c}(t)}{dt}$ of the particle, determine when the particle is ever at rest (momentarily). Also, when is the velocity vector ever vertical?

15. Using chain rule discussed on Friday, compute how the "altitude" $f(x, y)$ changes as you traverse the curve $\vec{c}(t)$ over time t. (i.e., compute $\frac{df(\vec{c}(t))}{dt}$).

(a.) $f(x, y) = xy$, $\vec{c}(t) = (e^t, cos(t))$

(b.) $f(x, y) = e^{xy}$, $\vec{c}(t) = (3t^2, t^3)$