MITE^{2}S 2008 : Physics III - Oscillations and Waves :: Problem Set 0.

Massachusetts Institute of Technology
Instructor: Hyun Youk
TA: Louis Fouché
(Due on Wednesday, June 25 at 1:20 PM in class.)

This problem set is designed to help you review some concepts from Newtonian mechanics. Mainly, this problem set requires you to use the following concepts:

- Newton's laws of motion.
- Work - energy theorem.
- Kinetic and potential energies.
- Breaking force into its vectorial components (horizontal, vertical forces).
- Impulse and momentum.

This problem set also introduces you to the usage of calculus in solving physics problems. Namely, it introduces the method of breaking down a macroscopic object into many segments of infinitesimal length and mass, which is one of the key technical tools we will be using in our course and is widely used in physics. Any time you see quantities such as dx and dm, you're working with infinitesimal quantities. Adding them up is what you're doing when you're performing integration.

General Note about Problem Sets in Physics III: First of all, don't let the apparent length of questions in the following pages fool you! All the questions in this problem set require only a few lines of simple calculations (usually 3-5 lines). Questions may seem wordy because I wanted to explain the physics being studied as explicitly as I could in words. But the actual solutions to any of these questions are quite short as mentioned above.

In many of the questions, the final answer is given to you. You will find in college level physics classes, this is often the case. The reason is that we're interested in seeing **how** you derive your final answer (what laws of physics are you using, what physical principles are you using, what intuition are you using, etc.) rather than the final answer itself. Indeed, if you write down just the final answer without showing us how you derived the result, you will get very little points! This doesn't mean that we want you to write long essays for each question: just show us all the work you had to do to actually get the result, nothing more, nothing less.

Finally, please let me (Hyun) or Louis know if you're having trouble with any of the problems here. Also, don't worry too much if the problems seem daunting! A course on waves and oscillations is typically taught in the sophomore year in college (including at MIT). Just remember this whenever you're having trouble! Please don't ever hesitate to contact me or Louis with questions.

Hyun's "Office" hours: My intention is to be at the Simmon's study halls from about 8 PM to 10 PM on Monday and Tuesday. I will try to drop by on the other days if I can. Please stop me and let me know after class if you'd like me to go over some topics with you in the evening at Simmons. Since I live on campus, this is actually quite easy for me to do. Of course, Louis will be around almost every evening to help you out as well.

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Problem 0. All about me! (Everyone gets 35 points!)

(i.) Tell us about yourself.

(ii.) What are your hobbies, favorite subjects in school, books, music, movies, etc.?

(iii.) What would you like to major in college? What are your career aspirations?

(iv.) Which of the following terms have you heard before? How comfortable are you with using them in math and physics classes?

- Taylor series (expansion).
- Differentiation and integration of simple functions
- Infinitesimal quantities.
- Momentum, potential and kinetic energy, impulse, angular momentum, Hooke's law for simple springs.

(v.) What would you like to learn in physics III?

(vi.) Is there a concept (or concepts) in physics (not necessarily related to oscillations and waves) that you'd like to learn about at some point? (Saying "No" is perfectly OK.). If so, what is it? Why are you curious about this particular topic (or topics)?

Problem 1. Adding up infinitesimal quantities: Integration (20 points)

Here are two short problems involving integration. The point in both of these problems is to let you see that integration is just a summation of a series of quantities, each of which is infinitesimal, but when added together gives you a finite (i.e. not infinitesimal) quantity. This sounds more abstract than it should, so let's work through a couple of examples:

Dust collector: Excited by the impending six weeks of adventures that await you in the MITES 08 program, you and your new friends have been running and jumping around the Simmons dormitory. Not at all impressed by this, your uptight physics TA (OK, your uptight physics III instructor) has decided to make you clean up all the dust particles that are now floating around in various corridors. To do this, he has given you a large sticky tape (flat sticky sheet of surface area A) which you will use to sweep across the various corridors.

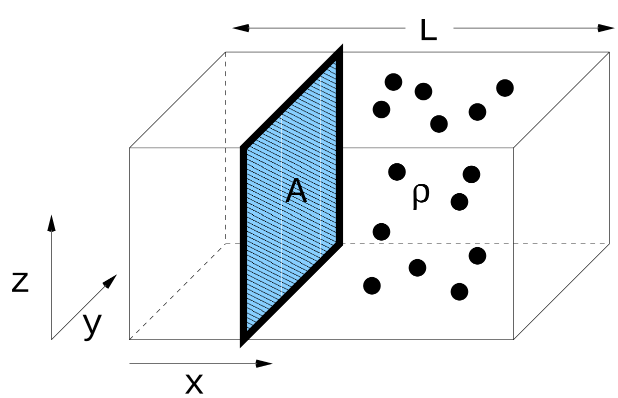

FIG. 1: Dust collector: Figure for problems 1 (a) and (b).

(a.) In one corridor, there's a *uniform* number density of dust particles suspended in the air. That is, the number of dust particles per volume (call it ρ) is the same throughout the entire stretch of the corridor of length L. If you take the flat sticky tape, hold it upright, and walk from one end of the corridor to the other end, what is the total number of dust particles that will be stuck to the surface of your tape at the end of your walk? Figure 1 shows the situation. Assume that the dust particles are suspended in the air, not moving at all. Your answer should be in terms of ρ, L, and A. This problem can be done without using integration.

(b,) In another corridor, the number density of dust particles (per volume) suspended in the air is not uniform, but instead is a function of their position in the corridor. In particular, it is given by $\rho(x) = kx$, where k is a constant and x is the position of the dust particle along the x-axis of the corridor. The left end of corridor is the origin ($x = 0$) while the right end is $x = L$. Notice that there is no density variation along the y and z axes. What is the physical unit of k? (i.e. is it length/time? mass/time? 1/length? or something else (if so, what is it)?). Suppose you repeat the cleaning exercise with your sticky tape in this new corridor, as you did in (a). Show that the total number of dust particles that are stuck to the surface of the tape after you've traversed the entire length L of the corridor is $\frac{kAL^2}{2}$. Again, assume that the dust particles themselves are not moving at all and that the tape is held at an upright position while you're walking down the corridor. Figure 1 shows the situation. Notice that I've given you the final answer here: the point in all of these problems is to show how you **derive** the final answer through step-by-step physical and mathematical reasoning.

Problem 2. Pulling two masses attached to a string. (45 points)

Two blocks are attached to a *massless* string of length $2L$ and are initially resting on a frictionless table. You grab the mid point of the string and pull it with a constant force T_0 to the right. Initially, the system (consisting of two blocks and the massless string) looks like Fig. 2(a). After some time, the system looks like Fig. 2(b). 2θ denotes the angle spanned by the string while the blocks are in transit. After sufficient amount of time of pulling, the two blocks come together and stick to each other, as shown in Fig. 2(c). Notice that the angle θ is thus a function of time and starts from $\theta = \frac{\pi}{2}$ in Fig. 2(a) and ends with θ=0 in Fig. 2(c). Throughout the entire motion, you're constantly exerting force T_0 to the right.

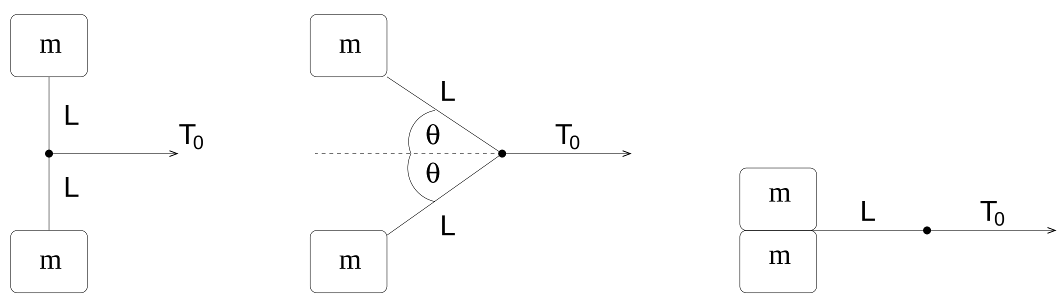

FIG. 2: (a) Left. (b) Middle. (c) Right.: Snapshots of the two blocks in transit. As your hand pulls the midpoint of the string at constant force T_0, the vertical distance between two blocks decreases (b), and eventually the blocks stick to each other (c).

To help you answer questions (a) - (d), you should draw a *free body diagram* showing all the forces acting on the two blocks, and the midpoint of the string. Start with Fig. 2(b) and draw in all the forces.

(a). What is the net force acting on the midpoint of the *massless* string at any instant? Justify your answer.

(b). Using your answer to (a), what must be the tension in the part of the string that's connected to the upper block? What is the tension in the part of the string that's connected to the lower block? Your answer should be in terms of θ and T_0. Hence the tension in both parts of the string continuously changes while the blocks are being pulled.

(c). Going back to your answer to (b), what force is acting vertically on each block in Fig. 2(b)? Your answer should be in terms of θ and T_0. From your answer, you should see that as θ approaches $\frac{\pi}{2}$, the tension in the string approaches infinity. Does this mean that in Fig. 2(a), the tension in the string is indeed infinite? If so, why is it that the two blocks don't initially have an "infinite" acceleration vertically, towards each other?

(d). What is the horizontal force acting on the two blocks in Fig. 2 (b)? Your answer should be *independent* of θ.

(e) and (f): (Corrected on Monday, June 23, 08 – Now combined into one problem): See our course web page for details: http://web.mit.edu/hyouk/www/

Problem 3. Falling rope. (Extra credit: Up to 30 points)

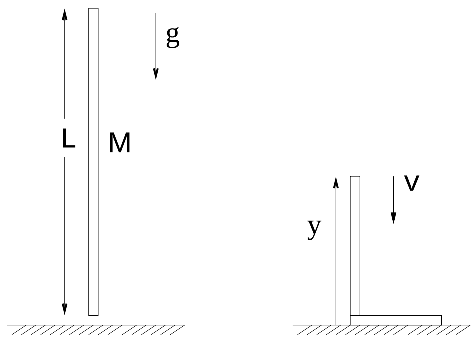

FIG. 3: (a) Left. (b) Right.: (a) shows the rope just before it is released. Some time after the release, a snapshot of the falling rope would look like (b); part of the rope is resting on the floor while the remaining part is falling down with speed v.

A rope of *uniform mass density* (i.e. mass per length is a constant) has length L and total mass M. You hold one end of the rope and suspend it in the air so that the other end of the rope is just above the ground, as shown in Fig. 3(a). You then release the rope and let it fall to the ground under the influence of gravity. The ground is actually equipped with a scale that tells you the weight felt by the ground due to the falling rope hitting the ground and stacking up. In this problem, we want to figure out how this scale reading is changing as the rope is falling down. Assume that g is the acceleration due to gravity and y is the distance between the top end of the rope and the floor.

(a). The rope is of uniform mass density (mass per length). What is this *linear mass density* λ? It should have dimension of mass per length.

(b). What is the scale reading (i.e. weight felt by the floor) just immediately after top of the rope has hit the ground? There are two methods that I know of for solving this problem. One involves looking at the rope's center of mass, while the other one is more "physical" (at least to me). You may solve this problem anyway you like (other than these two methods if you know of one), but here's how you might proceed: The following is the "physical" method:

(i.) Imagine dividing up the rope into many segments, each with an infinitesimal length dy. The rope is then all of these infinitesimal "blocks" strung together. What is the mass dm of one such infinitesimal segment of length dy? The linear mass density λ you found in (a) should be helpful here.

(ii.) Just immediately before the infinitesimal length segment located at the top of the rope hits the ground, show that the velocity of this mass segment is $v = \sqrt{2gL}$ downwards. For a full credit, you must justify, step-by-step, using physical reasoning how you arrived at this result. (*Hint*: Use **conservation of energy**).

(iii.) Just immediately after hitting the ground, the infinitesimal mass segment mentioned in (ii) comes to an immediate halt due to an upward force exerted by the floor. Actually, what we really mean by this is that the floor exerts an upward force F for a very short (infinitesimal) time interval dt on the mass segment, after which the mass segment comes to a halt. By relating the momentum of the mass segment just before hitting the ground, with the **impulse** delivered by the floor (via force F), show that $F = 2mg$.

(iv.) The floor must exert an upward force on the rope so that it can support the weight of the portion of rope that's already resting on the ground, while also exert an upward force to counteract the impact of the portion of the rope that's crashing to the ground. Thus, using your result of (iii), and by figuring out the weight of the rope that's already resting on the ground, show that the scale should read $3mg$ immediately after the top of the rope has hit the ground.

We have just derived the surprising result that while the weight of the rope is just mg, the impact of tiny (infinitesimal) portion of the rope at the top end of rope is $2mg$, thus imparting a force to the floor that's three times larger than the rope's own weight.