NB1140: Physics 1A - Classical mechanics and Thermodynamics
Problem set 4 - Rotational motion, torque, angular momenta, oscillations, and waves
Week 7: 9 - 13 January 2017

Problem 1. Kroket VS bitterball

At the Aula, you buy a bitterball and a kroket. It's a special day at the Aula's canteen: The bitterball is a perfect sphere of radius R and the kroket is a perfect cylinder of radius R and length L. Both have the same mass M that is uniformly distributed throughout their body.

(**a**) Calculate the rotational inertia I_k of the kroket with its axis of symmetry being the rotational axis (i.e. think of a toilet paper roll that rotates through the axle that holds the toilet paper).

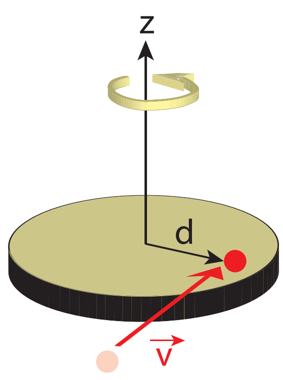
(**b**) Calculate the rotational inertia I_b of the bitterball with the rotational axis through its center.

(**c**) You simultaneously release the kroket and the bitterball from rest at the top of an inclined ramp. The ramp is inclined at an angle θ. There is no friction on the ramp. The kroket and the bitterball slide down the ramp without rolling at all. Which one arrives at the bottom of the ramp first?

(**d**) You simultaneously release the kroket and the bitterball from rest at the top of an inclined ramp. The ramp is inclined at an angle θ. Now the kroket and the bitterball roll down the ramp without slipping. Which one arrives at the bottom of the ramp first? Justify with calculations.

Problem 2. A small insect landing on a spinning disk
Part (a) of this problem will appear on Quiz 3

A turntable (circular disk) has rotational inertia I and is rotating with angular speed ω about a frictionless vertical axis (z-axis) (see figure). Viewed from the top, the turntable is rotating in the counterclock direction. In other words, the angular velocity vector of the turntable points in the +z direction. A small (point-like) insect of mass m, shown as the red dot in the figure, jumps onto the turntable and sticks at a point that is distance d from the rotation axis. The insect hits horizontally with its velocity $\vec{v}$ at 90 degrees ($\frac{\pi}{2}$ radians) angle to the turntable's radius, and in the same direction as the turntable's rotation (i.e., the radial vector of length d shown in the figure is perpendicular to the red



velocity vector $\vec{v}$ in the figure. $\vec{v}$ is the velocity of the insect just before it sticks to the disk at the point shown).

(a) For what speed $v = |\vec{v}|$ does the turntable's angular speed drop from half of its initial value?

(b) For what speed v will there be no change in the turntable's angular speed?

(c) For what speed v will the angular speed double?

Problem 3. Moving on a merry-go-round

A cockroach of mass m lies on the rim (i.e., the edge) of a circular disk with a uniform mass density. The disk has mass $10m$. The disk rotates freely about its center like a merry-go-round. Initially the cockroach and the disk rotate together with an angular velocity of 0.25 rad / s. Then the cockroach walks halfway to the center of the disk.

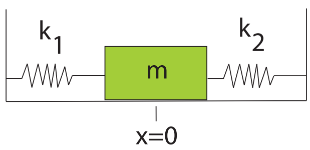
(a) What then is the angular velocity of the cockroach-disk system?

(b) Let K be the new kinetic energy (after the cockroach has moved into a point halfway to the center of the disk). Let K_O be the initial kinetic energy (when the cockroach was standing on the rim of the disk). Calculate $\frac{K}{K_O}$.

(c) What accounts for the change in the kinetic energy (i.e. why are K and K_O different?)

Problem 4. Two springs and a block

Consider a block of mass m connected to two massless springs, one on each side of the block, as shown in the figure. There is no friction in the system. The springs have Hookian spring constants of k_1 and k_2 respectively.



(a) Derive the equation of motion (equation for the acceleration of the block) for this system.

(b) As a function of the block's position $x(t)$ and velocity dx/dt, what is the total energy of the system?

(c) What is the maximum speed that the block can reach?

(d) Suppose that there is friction between the block and the floor. The coefficient of kinetic friction is μ_k. What is now the equation of motion?

(e) Suppose the mass of the block is doubled to $2m$. What is the frequency (number of oscillations / time) of the oscillating block of mass $2m$ compared to the frequency of the block with mass m?

(f) Suppose you replace the spring with the spring constant k_1 with another spring. This new spring consists of two shorter springs that are glued together. One shorter spring has spring constant k_a. You glue its left end to the left wall in figure . You glue its right end to another spring that has a spring constant k_b. You then attach the right end of the spring with spring constant k_b to the block of mass m (we set the mass back to m. It's no longer $2m$). What is now the frequency of oscillation (number of oscillations / time) of the block? The right end of the block is still attached to the spring with spring constant k_2.

Problem 5. Harmonic oscillator

A block of mass m is suspended by a spring of spring constant k. At $t = 0$ you hit the block such that its velocity becomes v_0. The block will start oscillating. Let $x(t)$ be the position of the block, $v(t)$ its velocity and $a(t)$ its acceleration at time t. The equation describing the motion of the block is given by

$$ma(t) + \gamma v(t) + kx(t) = 0,$$

where the first term is the inertial term from Newton's second law, the second term accounts for a damping force set by the viscous damping constant γ, and kx is the force exerted by the spring.

(**a**) Show that $x(t) = e^{\lambda t}$ is a solution and find an equation for λ using the given quantities.

You should have found in a) two possibilities for λ (except for one special case where there is only one value for λ). In case of two possible values for λ, the solution is given by a linear combination or superposition of the two solutions:

$$x(t) = Ae^{\lambda_+ t} + Be^{\lambda_- t}$$

We are now going to explore this solution in two different regimes. Take $x = 0$ and $v = v_0$ as initial conditions throughout the rest of the question.

(**b**) First, we consider a small damping force ($\gamma < 2\sqrt{mk}$). Find and sketch the solution. What is the frequency of the oscillation? Hint: Use Euler's formula ($e^{i\phi} = \cos(\phi) + i\sin(\phi)$) to simplify your equation.

(**c**) Now consider a large damping force ($\gamma > 2\sqrt{mk}$). Find and sketch the solution.

(**d**) The special case is called critical damping and it occurs when $\gamma = 2\sqrt{mk}$. Here the solution is given by $(A + Bt)e^{\lambda t}$. Find and sketch the solution for this case.

Problem 6. Waves in a string

A rope is attached to a wall. You start generating waves by swinging the loose end up and down. At the other end, the waves bounce back. The two waves are described by $u_1(x,t) = A\cos(kz - \omega t)$ and $u_2(x,t) = A\cos(kz + \omega t)$ respectively. Find an expression for the resulting wave. What type of wave is this?