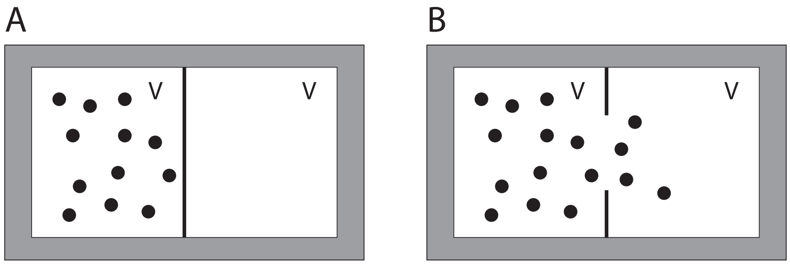
NB1140: Physics 1A - Classical mechanics and Thermodynamics
Problem set 5 - Entropy, information, and thermodynamics
Week 9: 23 - 27 January 2017

Problem 1. Entropy of an ideal gas inside a box with two rooms
Part (a) of this problem will appear on Quiz 4



Suppose that a box initially has two rooms inside it. Both rooms have the same volume. Each room has volume V. The box has a total of N identical gas particles. The box is surrounded by a thick insulating wall. This means that no energy or particle can enter of leave the box. The total energy in the box thus remains E at all times.

(**a**) For the box shown in Fig. 2A, derive the entropy of the system step-by-step. Carefully explain each key step of your derivation.

(**b**) Now suppose the partition (door) that separates the two rooms opens up (Fig. 2B). After a long time, the gas particles occupy both rooms. All particles can go back in and out of each room. What is the change in the entropy, compared to the entropy that you calculated in (a)? You can use the formula for entropy of N particles with energy E in a box of volume V that we derived in class:

$$S = k_B N ln(V) + \frac{3N}{2} k_B ln(\frac{2E}{m}) + k_B ln(constant) \qquad (1)$$

Note that here, we're using the fact that N is very large so that $(3N-1)/2 \approx 3N/2$.

Note from your answer that the change in entropy is a positive number. This shows that the total entropy has increased as a result of opening the partition between the two rooms. Note that the increase in entropy that you calculate here corresponds to an increase in the amount of disorder or equivalently the amount of information I. This makes sense. Before we opened the partition between the two rooms, we knew that all the particles were in the left room (Fig. 2A). But after the partition opened, we no longer know which room each particle is in. This corresponds to an increase in the number of bits that we now need to specify the system. This corresponds to an increase in the unpredictability of the system, which we showed in the lecture to be a measure of information content of the box.

Problem 2. Expansion of an ideal gas

Consider a monatomic ideal gas consisting of N atoms in an initial volume V_0 at an initial temperature T_0. The gas is then slowly expanded to a final volume $V_1 = 7V_0$.

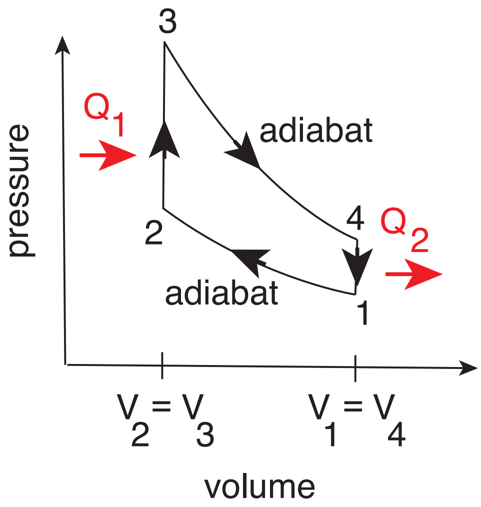
Calculate the temperature T_1, pressure P_1, the work W done by the gas, and the added heat$Q$, in case the expansion is

(**a**) Isothermal
(**b**) Isobaric
(**c**) Adiabatic

Compare and comment on the results.

Problem 3. The Otto cycle

The Otto cycle describes the working of internal combustion engines such as found in cars, lorries, and electrical generators. The cycle consists of the following processes (see figure):
1. First, the gas is adiabatically compressed from initial volume V_1 to volume V_2. 2. The gas is heated up by adding heat Q_1 while the volume is kept constant ($V_2 = V_3$). 3. Then the gas is expanded adiabatically to its initial volume ($V_4 = V_1$). 4. The gas is cooled down at constant volume as heat Q_2 is extracted from the gas.



(A realistic engine based on the Otto cycle contains two more processes but they will be neglected since they cancel each other out and hence do not contribute to any net changes).

(**a**) What is the efficiency of the engine in terms of the net work done W and/or the heat changes Q_1 and Q_2? (Note: you may not need all the variables)

(**b**) Compute the work done on the gas as (i) it is compressed adiabatically from 2 to 1, (ii) expanded adiabatically from 3 to 4.

(**c**) Show that the efficiency of the Otto cycle is 1 - $r^{1-\gamma}$, where $r = V_1/V_2$ is the compression ratio.