NB1140: Physics 1A - Classical mechanics and Thermodynamics
Solution to Quiz 3
Wednesday 18 January 2017

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The key in this problem is that the total angular momentum of the system (system = disc + insect) is conserved. This means that the total angular momentum of the system before the bug lands on the disc is the same as the total angular momentum of the system after the bug lands on the disc. Before landing on the disc, the total angular momentum of the system is

$$\begin{aligned}\vec{L}_{before} &= \vec{L}_{bug} + \vec{L}_{disc} \\ &= mvd\hat{z} + I\omega\hat{z} \\ &= (mvd + I\omega)\hat{z}\end{aligned} \tag{1a}$$

After the bug lands, the disc and the bug spin together with angular velocity ω_f. The total angular momentum of the system after the bug lands is

$$\begin{aligned}\vec{L}_{after} &= \vec{L'}_{bug} + \vec{L'}_{disc} \\ &= md^2\omega_f\hat{z} + I\omega_f\hat{z} \\ &= (md^2 + I)\omega_f\hat{z}\end{aligned} \tag{2a}$$

By conservation of total angular momentum, we have

$$\begin{aligned}(mvd + I\omega) &= (md^2 + I)\omega_f \\ \implies \omega_f &= \frac{mvd + I\omega}{md^2 + I}\end{aligned} \tag{3a}$$

Now, to have $\omega_f = \frac{\omega}{2}$, we need

$$\begin{aligned}\frac{\omega}{2} &= \frac{mvd + I\omega}{md^2 + I} \\ \implies v &= \frac{\omega}{2}(d - \frac{I}{md})\end{aligned} \tag{4a}$$