NB1140: Physics 1A - Classical mechanics and Thermodynamics
Final examination
Wednesday 1 February 2017
9:00h - 12:00h
(3 hours)

Total = 60 points

Instructions and tips:

1. The only aid you're allowed is a non-graphing calculator.
2. This exam has 6 problems.
3. Use a different answer book for each problem (so 6 different answer books).
4. Write your name, student number, and problem number on every answer book.
5. Except for a few parts of some problems, almost every part of every problem requires only short calculations (1-5 lines of calculations). If you find yourself doing a very long (pages) of calculation for one part of a problem, most likely you are on the wrong track. Stop and think.
6. If the solution of one part of the problem requires an answer from the previous part of the problem that you could not solve, then assign a variable to the answer and use it to obtain the formula for the next part of the problem (e.g. if you need answer for (b) to solve (c) but you could not solve (b), then assume you have α in (b), then use α in your calculations in (c)).
7. If you are stuck on one part of a problem for some time, move onto the next part of the problem or to a different problem. Don't spend all your time on just one problem.

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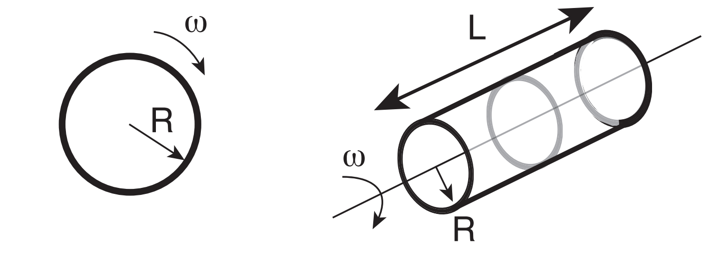
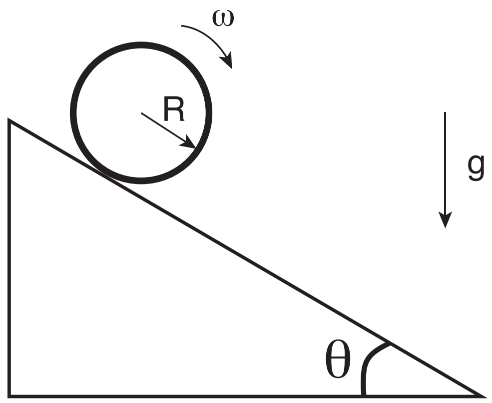
Problem 1. [Total = 12 points] - Onion ring and a hollow sausage

At the Aula, you buy a fried onion ring and a cheap sausage that is hollow at the Aula's canteen. The onion ring is a circle of radius R with a negligible thickness (i.e. it's a thin line bent into a circle) and mass M_o that is uniformly distributed (i.e. it has a uniform mass density). The sausage is hollow cylinder (i.e. no mass inside, like an emtpy toilet paper roll) of length L, radius R, and mass M_s that is uniformly distributed.



(A) Onion ring (B) Hollow cylindrical sausage



Moving down a ramp

(**a**) [**2 points**] By calculating the rotational inertia I_o of the onion ring with its axis of symmetry going through the center, show step-by-step that $I_o = M_o R^2$. Recall that rotational inertia I of any object is given by $I = \int dm \cdot r^2$, where the integral is performed all over the body of the object and r is the distance of mass element dm from the axis of rotation.

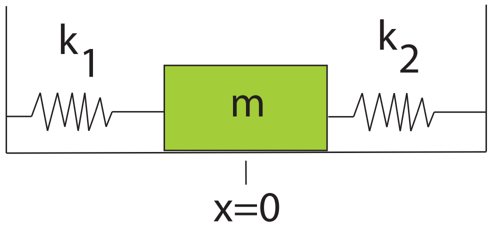
(**b**) [**3 points**] By calculating the rotational inertia I_s of the sausage with the rotational axis through its center (i.e. think of a toilet paper roll that rotates through the axle that holds the toilet paper), show step-by-step that $I_s = M_s R^2$.

(**c**) [**3 points**] Assume that $M_s > M_o$ (i.e. sausage has more mass than the onion ring). You simultaneously release the onion ring and the sausage from rest at the top of an inclined ramp. The ramp is inclined at an angle θ. There is no friction on the ramp. The onion ring and the sausage slide down the ramp without rolling at all. By calculating the linear acceleration of each object, determine which one arrives at the bottom of the ramp first.

(**d**) [**4 points**] Assume that $M_s > M_o$ (i.e. sausage has more mass than the onion ring). You simultaneously release the onion ring and the sausage from rest at the top of an inclined ramp. The ramp is inclined at an angle θ. Now the onion ring and the sausage roll down the ramp without slipping. By calculating the linear acceleration of each object, determine which one arrives at the bottom of the ramp first.

Problem 2. [Total = 14 points] - Simple harmonic motion

Consider a block of mass m connected to two massless springs, one on each side of the block, as shown in the figure. There is no friction in the system. The springs have Hookian spring constants of k_1 and k_2 respectively. Let $x = 0$ be the equilibrium position. You pull the block to one side (say to the right) by a distance A, then release the block. Let $x(t)$ be the position of the block at subsequent time t.



(**a**) [**2 points**] Derive the equation of motion (i.e. write down the Newton's 2nd law) for the block. Leave your answer in terms of d^2x/dt^2, x, and other constants given in this problem (i.e. No need to solve for x(t) and then plugging it into d^2x/dt^2 and $x(t)$).

(**b**) [**2 points**] As a function of the block's position $x(t)$ and velocity dx/dt, what is the total energy of the system? Leave your answer in terms of dx/dt, x, and other constants given in this problem. (i.e. No need to solve for x(t) and then plugging it into dx/dt and $x(t)$).

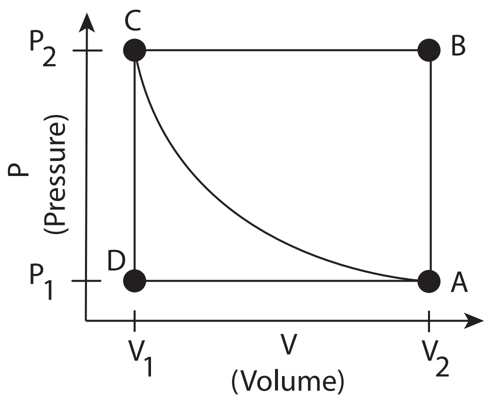
(**c**) [**2 points**] What is the maximum speed that the block can reach?

(**d**) [**2 points**] Suppose that there is friction between the block and the floor. The coefficient of kinetic friction is μ_k. What is now the equation of motion? (i.e. As in (a), write down Newton's 2nd law for the block and leave your answer in terms of d^2x/dt^2 and $x(t)$).

(**e**) [**2 points**] Suppose the mass of the block is doubled to $2m$. What is the frequency (number of oscillations / time) of the oscillating block of mass $2m$ compared to the frequency of the block with mass m? [Hint: angular frequency is not the same as the frequency]

(**f**) [**4 points**] Suppose you replace the spring with the spring constant k_1 with another spring. This new spring consists of two shorter springs that are glued together. One shorter spring has spring constant k_a. You glue its left end to the left wall in figure. You glue its right end to another spring that has a spring constant k_b. You then attach the right end of the spring with spring constant k_b to the block of mass m (we set the mass back to m. It's no longer $2m$). What is now the frequency of oscillation (number of oscillations / time) of the block? The right end of the block is still attached to the spring with spring constant k_2.

Problem 3. [Total = 10 points] - Ideal gas cycle



Shown in the figure is a pressure-volume diagram for a box of ideal gas. The curved path joining point C with point A corresponds to an isothermal path (i.e., path along which the temperature remains constant). Along the isotherm, the ideal gas has temperature T_O. For this ideal gas, we define $\gamma = \frac{C_p}{C_v}$. C_p is the molar specific heat at constant pressure. C_v is the molar specific heat at constant volume. The ideal gas law states that $pV = nRT$, where p is the pressure on the walls of the box, V is the volume of the box, n is the number of moles of gas in the box, R is the universal gas constant, and T is the temperature of the gas. Recall that $C_p = C_v + R$ and that the change in the total (internal) energy ΔE of ideal gas is $\Delta E = Q + W$ where Q is the heat and W is the work involved in a process.

(**a**) [**3 points**] Calculate the total work done on the gas as it goes around the cyclic path ABCA (i.e. starts from A, then to B, then to C, then to A).

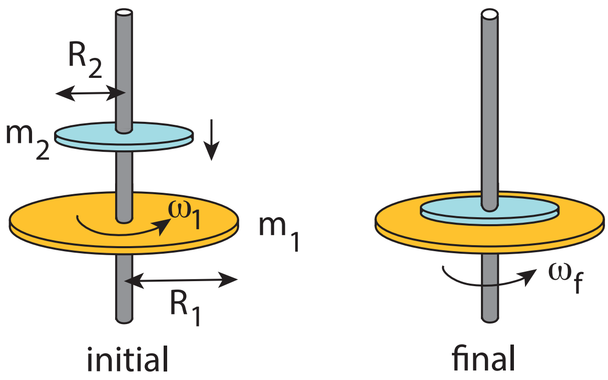
(**b**) [**2 points**] How much heat enters or exits the box during the process from point A to point B? Indicate if the ideal gas gains heat or loses heat in going fro A to B.

(**c**) [**3 points**] Calculate the net work done on the gas as it goes around the cyclic path ACDA.

(**d**) [**2 points**] How much heat enters or exits the box in going from C to D? Indicate if the ideal gas gains heat or loses heat in going from C to D.

Problem 4. [Total = 9 points] - Collision between two discs

In the figure, the lower disk of mass m_1 and radius R_1 is rotating at angular speed ω_1 in the direction as shown. It is rotating on a frictionless axle. The upper disk of mass m_2 and radius R_2 is initially not rotating. It drops freely down through the axle and lands on the lower disk. Frictional forces between the lower and upper disks bring the two disks to a common rotational speed ω_f. Both disks are infinitesimally thin (i.e. approximately zero thickness) and have uniform mass densities.



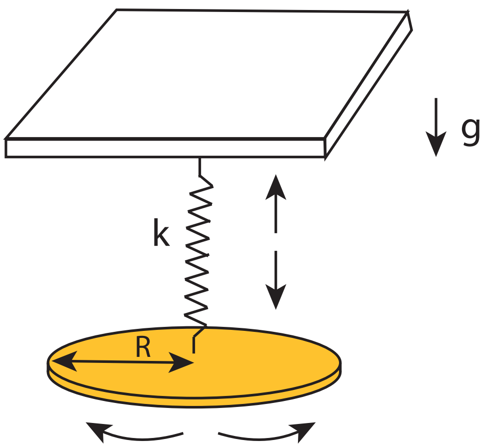
(**a**) [**3 points**] By step-by-step calculation, show that the rotational inertia I of a uniform disc of radius R and mass M that spins about the axle through its center (as shown in the figure) is $I = MR^2/2$. [Hint: Break up the disc into concentric rings, calculate the inertia of each ring, then sum up all the rings].

(**b**) [**3 points**] What is the common rotational speed ω_f after the top disc lands on the lower disc and both discs spin together?

(**c**)[**3 points**] What are the initial and final kinetic energies? Why are these two energies not equal?

Problem 5. [Total = 8 points] - Oscillating up and down, and twisting back and forth.

A solid disk of radius R, mass M, and rotational inertia I about the axis that vertically goes through its center is suspended from a ceiling by a spring with a spring constant k and torsional constant γ. The spring is connected to the center of the disc.



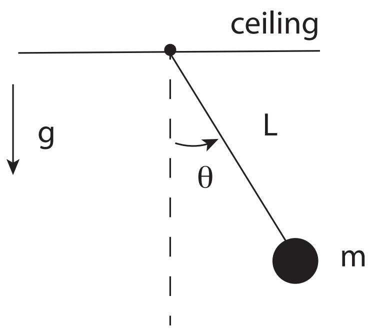
(**a**) [**2 points**] Ignoring the twisting (torsional) motion and assuming that the disk only oscillates up and down, write down the equation of motion (i.e. Newton's 2nd law equation) for the solid disk. Here, let $y = 0$ be the vertical equilibrium position of the disk and $y(t)$ be the vertical displacement above and below this equilibrium position at time t. Leave your answer in terms of d^2y/dt^2, y, and other constants given in this problem.

(**b**) [**2 points**] The side-to-side twisting (torsional) motion is described by the following equation of motion:

$$I\frac{d^2\theta}{dt^2} = -\gamma\theta \tag{1}$$

where $I = MR^2/2$ for the disk and θ is the twist angle relative to equilibrium position. In terms of k and γ, what value of R gives the same period for the vertical and torsional oscillations of the system?

(**c**) [**4 points**] A pendulum consists of a massless string of length L that connects a tiny (point-like) ball of mass m to a frictionless pivot at the ceiling (see figure). By step-by-step derivation of the equation of the motion for this simple pendulum and calculating the frequency of oscillations from it, show that the pendulum's frequency of small (simple harmonic) oscillation does not depend on the mass of the ball.



Problem 6. [Total = 7 points] - Basic properties of waves

(**a** $\sim$ **c**) A transverse wave propagates along a string. As a function of the horizontal position x, the height h of the string at time t is

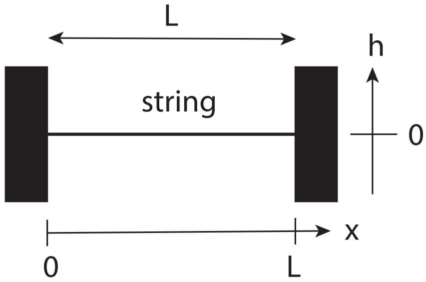
$$h(x,t) = Asin(kx + \omega t) \tag{2}$$

(**a**) [**1 point**] Which direction (towards +x or -x) does the wave propagate?

(**b**) [**1 point**] What is the wavelength in terms of the given constants?

(**c**) [**2 points**] Consider a fixed location $x = x_o$. You watch the height of the string at this fixed x position. With what frequency does this height oscillate? Is it a simple harmonic motion?

(**d**) [**3 points**] Now let's consider a different situation. We have two walls that are apart by distance L. A string has one end fixed to one wall and the other end fixed to the other wall. The figure shows the string at rest. Suppose you create a transverse standing wave. Draw two examples of such standing waves that have different wavelengths. Indicate the wavelength of each one. [Hint: Note that a transverse standing wave means



that the height $h(x)$ of the string at position x will take on special values when $x = 0$ or $x = L$].