Final examination - Potentially useful formulae
Wednesday 1 February 2016

Rotational motion

No need to remember formulas of various objects' rotational inertias. If necessary, you will be given these values in the problems.

Torque $\vec{\tau}$ about any reference point (origin) that you choose is

$$\vec{\tau} = \vec{r} \times \vec{F} \tag{1}$$

where $\vec{r}$ is the position vector from the origin to some point P, and $\vec{F}$ is the force acting at point P. For an object with rotational inertia I, the total total torque $\vec{\tau}_{\text{total}}$ acting on it and its angular acceleration $\vec{\alpha}$ are related by

$$\vec{\tau}_{\text{total}} = I\vec{\alpha} \tag{2}$$

If the object has angular velocity $om\vec{e}ga$, then its angular momentum $\vec{L}$ is

$$\vec{L} = I\vec{\omega} \tag{3}$$

The angular momentum and torque are related by

$$\tau = \frac{d\vec{L}}{dt} \tag{4}$$

A rolling object that has mass m, rotational inertia about its center of mass I_{cm}, and angular speed ω about its center of mass has the following kinetic energy KE

$$KE = \frac{mv^2}{2} + \frac{I_{cm}\omega^2}{2} \tag{5}$$

where v is the linear speed of the object's center of mass.

If a circular object of radius R rolls on the ground without slipping, then we have

$$R\omega = v \tag{6}$$

where v is the linear speed of the object's center of mass and ω is the rotational speed of the object about its center of mass.

Oscillations (Simple harmonic motion)

An object that undergoes a simple harmonic motion obeys the following equation of motion

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \tag{7}$$

where x is the object's position, t is time, and ω is the angular frequency of the oscillation. The frequency f is related to ω as

$$2\pi f = \omega \tag{8}$$

The period T of oscillation is $1/f$. The solution to the simple harmonic oscillator's equation of motion (equation (7)) is

$$x(t) = Acos(\omega t + \phi) \tag{9}$$

where A is the amplitude of the oscillation and ϕ is the phase of oscillation. For oscillatory systems, Hookian springs are often important. The potential energy stored in a Hookian spring with a spring constant k that has been compressed or stretched by distance z is

$$E_{\text{spring}} = \frac{kz^2}{2} \tag{10}$$

Wave motion

A simple harmonic wave is a sinusoidal wave that travels along the x-axis and described by the following

$$y(x,t) = Acos(kx - \omega t) \tag{11}$$

where A is the amplitude of the wave, k is the wave number, ω is the angular frequency, t is time, x is position along x-axis, and y is the position (height) along the y-axis. The wave number and the period T obey

$$k = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T} \tag{12}$$

where T is the period, λ is the wavelength, and ω is the angular frequency. The wave travels at speed v along the x-axis and is related to the wavelength, period, and frequency f by

$$v = \frac{\lambda}{T} = \lambda f \tag{13}$$

Thermodynamics

An ideal gas in a box of volume V obeys the following "ideal gas law":

$$pV = nRT \tag{14}$$

where p is the pressure of the gas (i.e. pressure that the gas exerts on the walls of the box), V is the volume of the box, n is the number of moles of the gas, R is a "universal gas constant" (a constant number equal to 8.314 J/K), and T is the temperature (in Kelvins) of the gas. Note that the total integer number N of gas particles inside the box is $N = nN_A$, where N_A is the Avogadro's number (6.02 x $10^2 3$).

The work W done *on* the gas when a volume of the box changes from V_1 to V_2 is

$$W = \int_{V_1}^{V_2} dW = -\int_{V_1}^{V_2} pdV \tag{15}$$

Note that $W > 0$ means that work has been done *on* the system (e.g. you do work on the gas) where as $W < 0$ means that the system (e.g. ideal gas) has done the work (e.g. gas did work on you).

According to the first law of thermodynamics, during a process that involves work W done *on* on the system and heat Q that enters ($Q > 0$) or leaves ($Q < 0$) the system, we have

$$\Delta E_{\text{process}} = W + Q \tag{16}$$

where $\Delta E_{\text{process}}$ is the change in the energy of the system (e.g. box of ideal gas) during the process.

An **isothermal process** occurs at constant temperature. This means that during the whole process, $dT = 0$ (i.e. when breaking up process into many infinitesimal processes, the temperature change dT in each infinitesimal process is zero).

In a **constant volume process**, $dV = 0$.

In an **isobaric (constant pressure) process**, $dp = 0$.

In an **adiabatic process**, $Q = 0$, where Q is the total heat that enters / leaves the box of gas during the process.

Heat of transformation L is the energy per mass (i.e. J / kg) required to change the phase of the matter. Thus the total energy (heat) Q that is required to change a system of mass m from one phase (e.g. liquid) to another phase (e.g. gas) is

$$Q = Lm \tag{17}$$

The **specific heat** c is the amount of energy (heat) Q that is required to increase the temperature of an object of mass m by an amount ΔT and is given by

$$Q = mc\Delta T \tag{18}$$