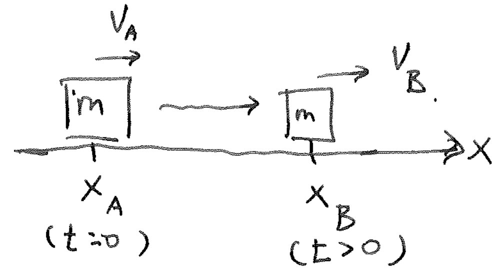
yesterday, we found (for 1-dim motion):



Work $_{A\to B}$: defined as $W_{A\to B} = \int_{x_A}^{x_B} F_{total}\, dx$

And from $\vec{F} = m\vec{a}$, we found that:

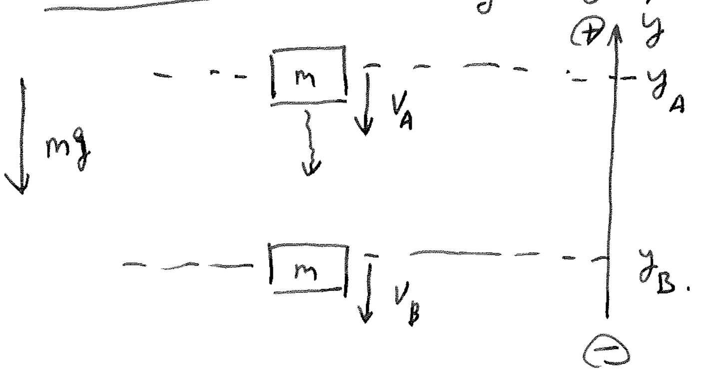
$$\boxed{\underbrace{\frac{mv_B^2}{2} - \frac{mv_A^2}{2}}_{\Delta KE} = W_{A\to B}}$$

ΔKE ← change in kinetic energy

$W_{A\to B}$ ← work done by net force in going from A to B.

special forces

1) Constant gravity: $F_g = mg$, then we found:



(so $y_A > y_B$)

$W_{grav} = \int_{y_A}^{y_B} F_{grav}\, dy$

W_{grav} ← work done by constant gravity

$= \int_{y_A}^{y_B} (-mg)\, dy$

(−mg) ← ⊖ because we chose ↑⊕ ↓⊖

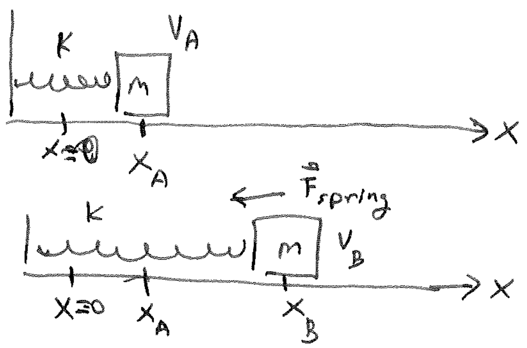
$= -mg\,(y_B - y_A)$

So: $$\boxed{\frac{mv_B^2}{2} + mg\,y_B = \frac{mv_A^2}{2} + mg\,y_A}$$

We define: $U_{grav} = mgy$ $\quad y \equiv$ vertical position

U_{grav} ← gravitational potential energy.

special force #2: spring force



$(t=0)$

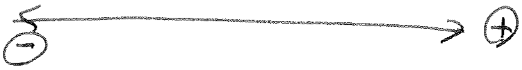
$(t>0)$

$$W_{A\to B} = \int_{x_A}^{x_B} F_{spring}\, dx$$

$$= \int_{x_A}^{x_B} -kx\, dx$$

$$= \left.\frac{-k\,x^2}{2}\right|_{x_A}^{x_B}$$

$$= \frac{-k x_B^2}{2} + \frac{k x_A^2}{2}$$



Important (with $x=0$ being the equilibrium position)

So:
$$\frac{m V_B^2}{2} - \frac{m V_A^2}{2} = W_{A\to B}.$$

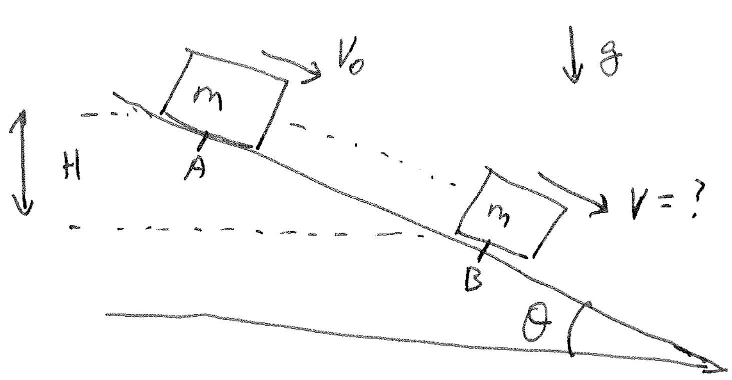
$$\boxed{\frac{m V_B^2}{2} + \frac{k x_B^2}{2} = \frac{m V_A^2}{2} + \frac{k x_A^2}{2}}$$

for this reason, we define:

$$\boxed{U_{spring} = \frac{k x^2}{2}}$$

where x = ~~distance of~~ displacement relative to equilibrium.

Example from yesterday:



A mass slides down a ramp.

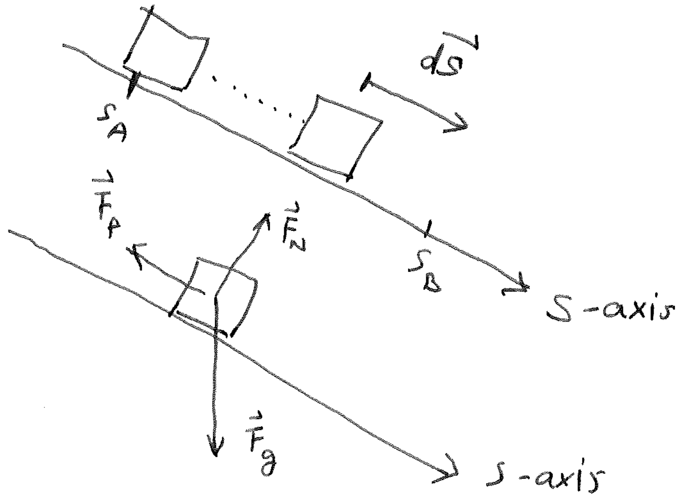
- There's constant gravity,
- coefficient of kinetic friction is μ_k.
- Block starts to move down at initial speed V_0.
- What is V, after the block goes down by distance H?

- yesterday, we solved this by calculating total force along the incline.

Today, a different method:

$$\frac{mV^2}{2} - \frac{mV_0^2}{2} = W_{A\to B}.$$

Now, $\quad W_{A\to B} = \displaystyle\int_{s_A}^{s_B} \vec{F}_{total} \cdot d\vec{s}$



$$= \int_{s_A}^{s_B} (\vec{F}_f + \vec{F}_g + \vec{F}_N) \cdot d\vec{s}$$

$$= \underbrace{\int_{s_A}^{s_B} \vec{F}_f \cdot d\vec{s}}_{= W_f} + \underbrace{\int_{s_A}^{s_B} \vec{F}_g \cdot d\vec{s}}_{= W_g} + \underbrace{\int_{s_A}^{s_B} \vec{F}_N \cdot d\vec{s}}_{= W_N}$$

$\vec{F}_f$ = friction

$\vec{F}_g$ = gravity

$\vec{F}_N$ = normal force.

so $\vec{F}_N \cdot d\vec{s} = |\vec{F}_N||d\vec{s}|\cos(\pi/2) = 0.$

$$= W_f + W_g + W_N$$

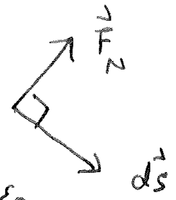
work done by friction

work done by gravity

work by normal force

so the work by ~~different~~ different forces just add up.

① $\underline{W_N}$:

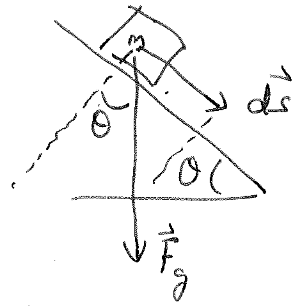


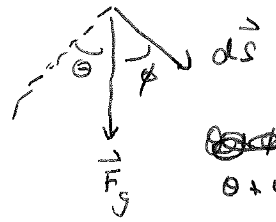
so $\vec{F}_N \cdot \vec{ds} = |\vec{F}_N|\, ds \cos(\pi/2)$
$= 0.$

$ds = |\vec{ds}|$

so $W_N = \int_{s_A}^{s_B} 0 = 0.$

② $\underline{W_g}$: $W_g = \int_{s_A}^{s_B} \vec{F}_g \cdot \vec{ds}$





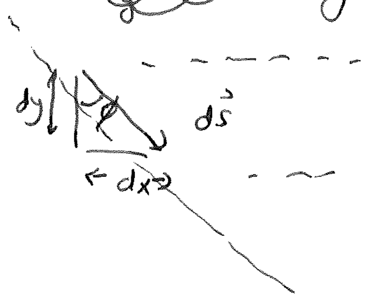
$\theta + \phi = \pi/2$

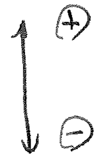
~~$\vec{F}_g \cdot \vec{ds} = mg\, ds \cos(\pi/2 - \theta)$~~

so $\vec{F}_g \cdot \vec{ds}$

$= -mg \underbrace{(ds)\cos\phi}_{''\,dy}$

$= mg\, dy$





so $W_g = \int_{s_A}^{s_B} \vec{F}_g \cdot \vec{ds}$

$= \int_{y_A}^{y_B} -mg\, dy$

$\vec{F}_g = -mg$

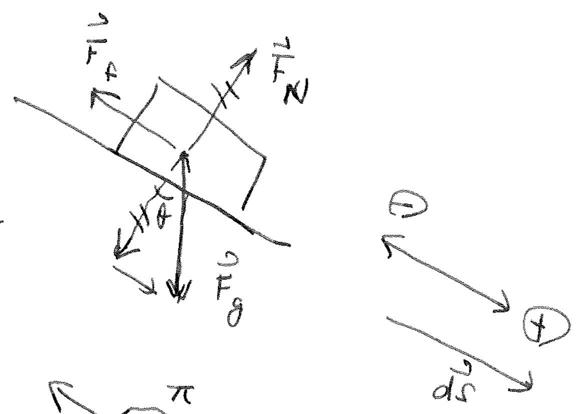
$= -mg(y_B - y_A)$ ⇒ $-(y_B - y_A) = H.$

$= mgH$

③ $\underline{W_f}$: $\int_{s_A}^{s_B} \vec{F}_f \cdot \vec{ds}$.

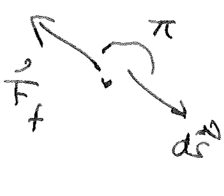
$|\vec{F}_f| = \mu_k F_N$

$= \mu_k mg\cos\theta$

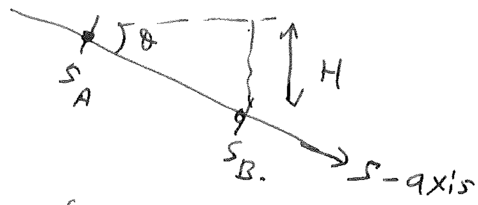


$= \int_{s_A}^{s_B} -\mu_k mg\cos\theta\, ds$

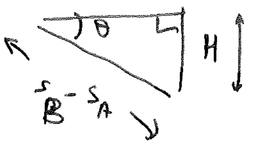
$= -\mu_k mg\cos\theta\,(s_B - s_A)$



so $\quad W_f = -\mu_k m g \cos\theta \, (s_B - s_A)$



but



$(s_B > s_A)$ ⊕

$$(s_B - s_A)^2 = H^2 + \left[(s_B - s_A)\cos\theta\right]^2$$

$$\Rightarrow (s_B - s_A)^2 \left[\underbrace{1 - \cos^2\theta}_{=\sin^2\theta}\right] = H^2$$

$$\Rightarrow (s_B - s_A)^2 = \frac{H^2}{\sin^2\theta} \Rightarrow s_B - s_A = \frac{H}{\sin\theta}$$

so $\quad W_f = -\mu_k m g \dfrac{H}{\tan\theta}$

so Putting everything together;

$$\frac{mv^2}{2} - \frac{mv_0^2}{2} = W_{total}$$

$$= W_N + W_g + W_f$$

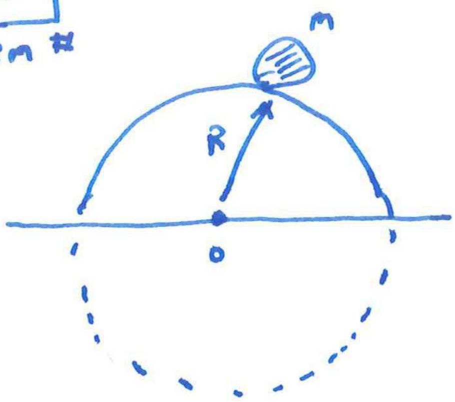
$$= mgH + W_f$$

$$\Rightarrow \boxed{\frac{mv^2}{2} = \frac{mv_0^2}{2} + mgH - \mu_k mg\frac{H}{\tan\theta}}$$

← from this we can solve for V.

Some problems from Mastering physics
(3rd edition of book).

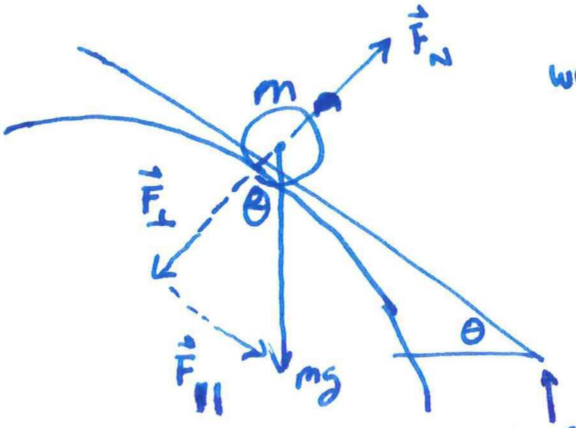
$\boxed{7.60}$
problem #



o ← center of circle

R = radius of circle.

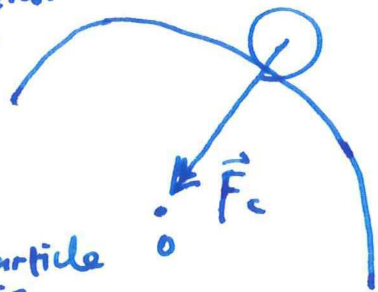
when moving in circle
(i.e. in contact with the sphere.)
we have <u>centripetal</u> <u>acceleration</u>. a_c.



tangent to circle where the particle is.

so $\vec{F}_c = \vec{F}_\perp + \vec{F}_N$

Centripetal force



So here, $|\vec{F}_N| \neq |\vec{F}_\perp|$

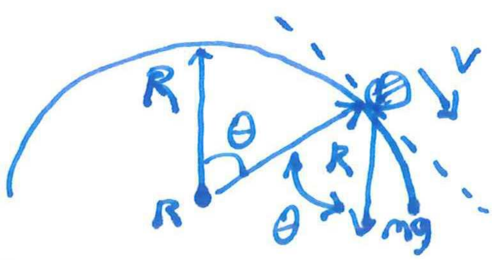
As V increases, $|\vec{F}_N|$ must decrease in order to increase $|\vec{F}_c|$. But eventually, $|\vec{F}_N| = 0$ and cannot decrease anymore. At this point, $|\vec{F}_c|$ is maximum possible value. For higher speeds, the $|\vec{F}_c|$ should be higher than this maximum value of $|\vec{F}_c|$. But since $|\vec{F}_c|$ cannot get any higher, the particle <u>cannot</u> go in circle anymore.

At this point, the particle leaves the hemisphere.

Here, we have: $|\vec{F}_c| = |\vec{F}_\perp| = mg\cos\theta$

maximum centripetal force

so $\frac{mV^2}{R} = mg\cos\theta \Rightarrow V^2 = Rg\cos\theta$

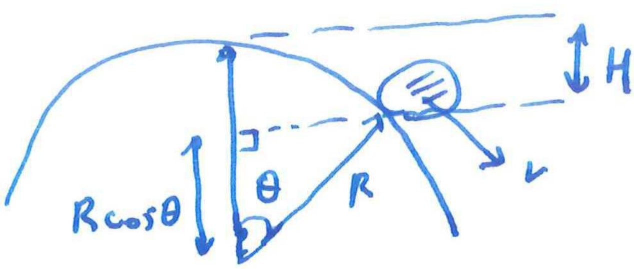


$$\left.\begin{aligned} W_{grav} &= mgH \\ &= \frac{mV^2}{2} \end{aligned}\right\} \Rightarrow V^2 = 2gH$$

work done by gravity

$$\Rightarrow R\cancel{g}\cos\theta = 2\cancel{g}H$$

$$\therefore \boxed{\frac{R\cos\theta}{2} = H}$$



But we don't know what θ is yet.

To find θ, note that $H + R\cos\theta = R$.

$$\Rightarrow \frac{R\cos\theta}{2} + R\cos\theta = R$$

$$\Rightarrow \frac{3R\cos\theta}{2} = R \Rightarrow \cos\theta = 2/3$$

Thus: $$H = \frac{R}{2}\cdot\frac{2}{3} = R/3.$$

So the particle leaves the hemisphere when $H = R/3$.

Problems

7.62 & 7.66, 7.55, 7.~~4~~5 ← same idea as 7.57

7.62: similar to example in lecture 3.

7.66: related to Problem set 2.

7.55: similar to examples in lecture 3 + 4 that involve friction.

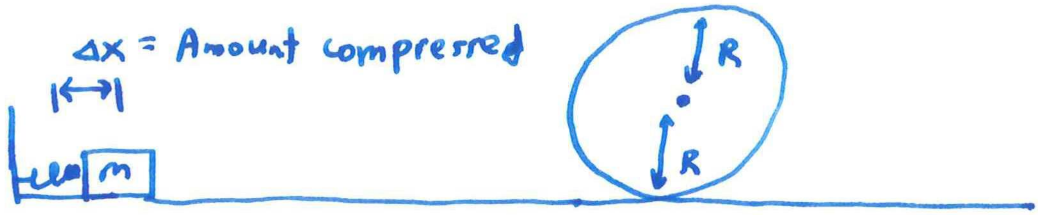
$\boxed{7.57}$ ~~Similar idea as in problem 7.60.~~

problem #

↳ (i.e. centripetal acceleration).

· Use your intuition first, like in problem 7.60 to think about normal force $\vec{F}_N$ and gravity.

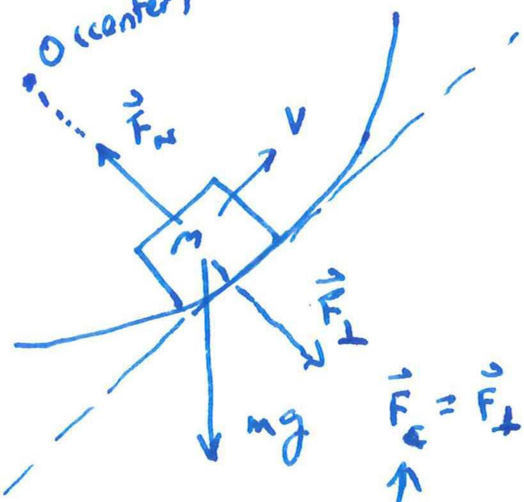
Δx = Amount compressed



Need to know minimum possible value of Δx.

At the top of the circle, m must have enough speed ~~From energy conservation~~. otherwise, it falls off.

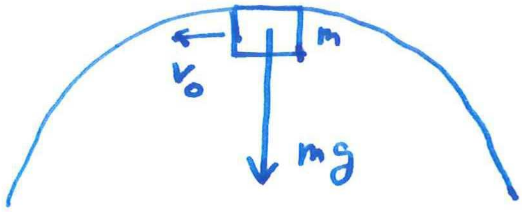
First, energy conservation: $\dfrac{k(\Delta x)^2}{2} = mg(2R) + \dfrac{mv_0^2}{2}$



$\vec{F}_c = \vec{F}_\perp + \vec{F}_N$

centripetal

Normal



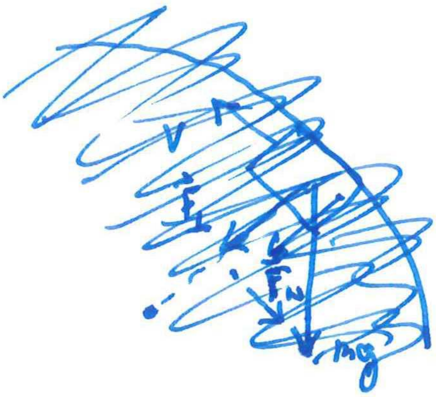
o ↖ center of circle.

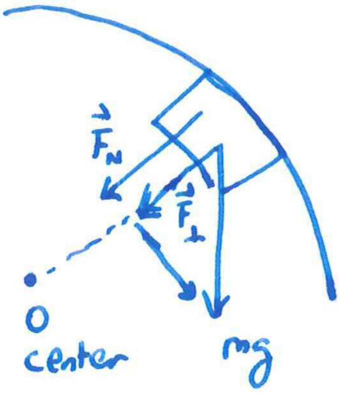
Here, the minimum possible centripetal force is

$|\vec{F}_c| = |\vec{F}_{gravity}|$

(with $|\vec{F}_N| = 0$.)

(because you cannot eliminate gravity)





so: $F_c = \dfrac{mv_0^2}{R} = mg$

$\Rightarrow v_0^2 = Rg$

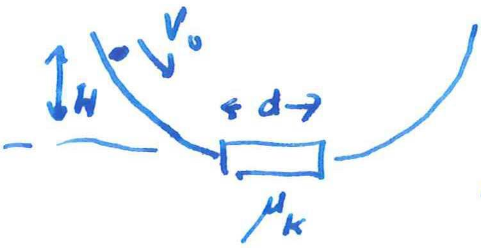
so: $\dfrac{k(\Delta x)^2}{2} = 2mgR + \dfrac{mRg}{2}$

$\Rightarrow \boxed{\Delta x = \sqrt{\dfrac{5mgR}{k}}}$ ☒

$\boxed{7.54}$ Problem #

Idea: ~~frictionn~~ friction always takes away energy.

(i.e. Work done by friction always takes energy away.)



v_0 = initial speed.

H = initial Height

$$E_{initial} = \frac{mv_0^2}{2} + mgH$$

$$W_{friction} = -\mu_k mgL$$

work done by friction.

$L \equiv$ total distance ~~on the frictionless~~ travelled on friction surface.

Since friction takes away energy:

(so work done on block is negative)

(remember: $W = \Delta E$)

change in total energy.

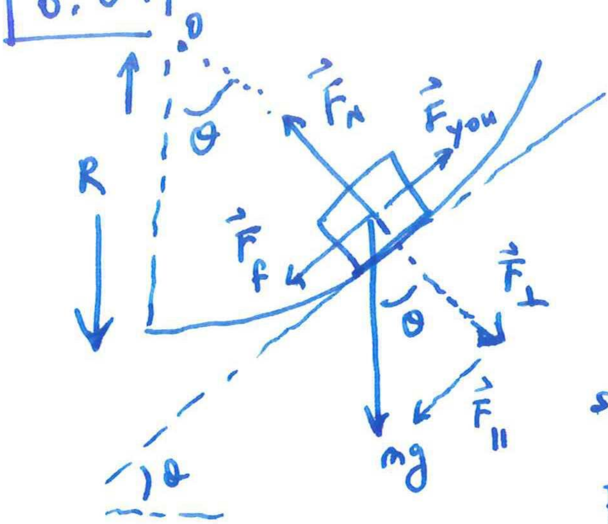
$$\mu_k \cancel{m} g L = \frac{\cancel{m}v_0^2}{2} + \cancel{m}gH$$

$$\Rightarrow L = \frac{\left(\frac{v_0^2}{2} + gH\right)}{\mu_k g}$$

$\Downarrow$

$\frac{L}{d}$ = # times the block crosses the friction floor.

$\boxed{6.84}$ ← problem



$\vec{F}_{you}$ = force that you exert to push the block upwards

"moving very slowly" means that it's moving at $v \approx 0$. (and nearly no acceleration)

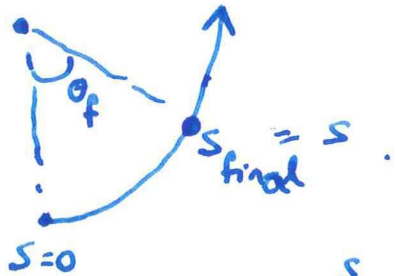
so all forces cancel at all times.

$F_f = \mu_k F_N$, so $|\vec{F}_N| = |\vec{F}_\perp|$

$= \mu_k mg \cos\theta$

The $\underline{\underline{\text{total}}}$ work you do to push block up to height h is:

$$W_{total} = \int_0^{s_f} \vec{F}_{you} \cdot ds$$



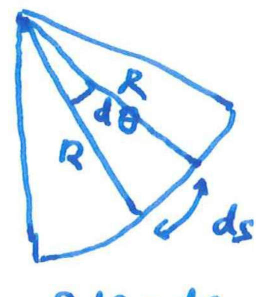
$$= \int_0^{s_{final}} (|\vec{F}_{\parallel}| + |\vec{F}_f|)\, ds$$

$$= \underbrace{\int_0^{s_{final}} |\vec{F}_{\parallel}|\, ds}_{= W_{grav}} + \underbrace{\int_0^{s_{final}} |\vec{F}_f|\, ds}_{= W_{fric}}$$

W_{grav} ↑ work done against gravity

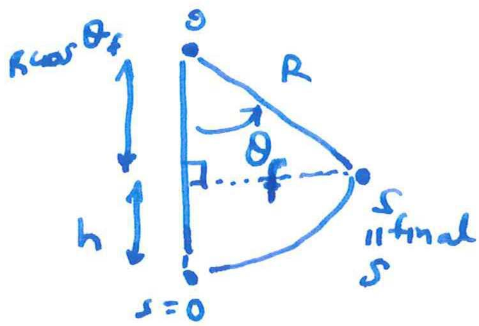
W_{fric} ↑ work done against friction.

$$W_{fric} = \int_0^{s_{final}} \mu_k mg \cos\theta\, ds$$



$\underline{R\,d\theta = ds}$

So
$$W_{fric} = \int_0^{s_{final}} \mu_k mg \cos\theta\, ds$$
$$= \int_0^{\theta_f} \mu_k mg \cos\theta\, R\, d\theta$$
$$= \mu_k mg R \sin\theta_f$$



$h + R\cos\theta_f = R$.

$$\Rightarrow \frac{R-h}{R} = \cos\theta_f$$

$$\Rightarrow \frac{(R-h)^2}{R^2} = \cos^2\theta_f$$

And $\sin^2\theta_f = 1 - \cos^2\theta_f$

$$= 1 - \frac{(R-h)^2}{R^2}$$

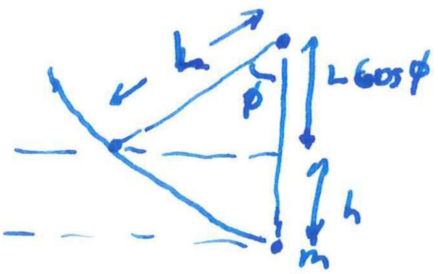
$$= \frac{\cancel{R^2} - \cancel{R^2} + 2Rh - h^2}{R^2}$$

$$= \frac{h(2R-h)}{R^2} \Rightarrow \sin\theta_f = \frac{\sqrt{h(2R-h)}}{R}$$

Hence: $\boxed{W_{friction} = \mu_K mg\sqrt{h(2R-h)}}$

$\boxtimes$

$\boxed{6.56}$ ← problem #



$$\Delta U_{grav} = mgh$$

Work you do $= mgh$ $\qquad h + L\cos\phi = L$

$$\Rightarrow h = L(1-\cos\phi)$$

$$\therefore \quad = mgL(1-\cos\phi)$$