NB1140: Physics 1A - Classical mechanics and Thermodynamics
Midterm exam
Wednesday 14 December 2016
9:00h - 12:00h
(3 hours)

Total = 50 points

Instructions and tips:

1. The only aid you're allowed is a non-graphing calculator.
2. This exam has 6 problems.
3. Use a different answer book for each problem (so 6 different answer books).
4. Write your name, student number, and problem number on every answer book.
5. Except for a few parts of some problems, almost every part of every problem requires only short calculations (1-5 lines of calculations). If you find yourself doing a very long (pages) of calculation for one part of a problem, most likely you are on the wrong track. Stop and think.
6. If the solution of one part of the problem requires an answer from the previous part of the problem that you could not solve, then assign a variable to the answer and use it to obtain the formula for the next part of the problem (e.g. if you need answer for (b) to solve (c) but you could not solve (b), then assume you have α in (b), then use α in your calculations in (c)).
7. If you are stuck on one part of a problem for some time, move onto the next part of the problem or to a different problem. Don't spend all your time on just one problem.

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Problem 1. [Total = 6 points] - Collisions and center of mass

A particle of mass m moves with a constant *speed* v ($v > 0$) towards a stationary particle of mass M on its right (Figure 1). The two particles collide perfectly elastically. You stand still on the ground and observe the collision. All motion occurs in one-dimension (along a line). After the collision, you note that the particle of mass m stops moving.

Figure 1: *1-dimensional collision: Before the collision*

(**a**) [**2 points**] What is the *velocity* of the particle of mass M after the collision? Express your answer in terms of m, M, and v (give both magnitude and direction since it's velocity).

(**b**) [**2 points**] How is M related to m, given that this is a perfectly elastic collision?

(**c**) [**1 points**] What is the *velocity* of the system's center of mass before the collision? Here, you can assume that the positive x-axis is in the same direction as the initial velocity of the particle of mass m (i.e. to the right).

(**d**) [**1 point**] Now you are on a bicycle that is moving at the same velocity as the center of mass. You observe the particles after the elastic collision. As measured by you, what is the total momentum of the system after the collision? Explain your answer.

Problem 2. [Total = 9 points] - Block sliding down a spherical surface

A block of mass m is initially sitting at rest on top of a frictionless, spherical surface of radius R. A very tiny push causes it to slide down the surface starting from rest. There is a constant gravitational force. The magnitude of the constant acceleration due to gravity is g ($g > 0$).

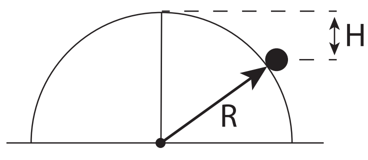

Figure 2: *Block of mass m sliding down on a spherical surface. The sphere has radius R. H is the height of the block below the top of the spherical surface.*

(**a**) [**2 points**] The block slides down and follows the circular path until it loess contact with the sphere at some point. While the block is moving on the sphere (i.e. before it loses contact with the sphere), there is centripetal force acting on it to keep it moving in the circular path. Calculate the magnitude of this centripetal force. Express your answer in terms of m, g, R, and the vertical distance H measured from the top of the sphere where the block was initially at.

(**b**) [**5 points**] Calculate the magnitude of the normal force that acts on the block at the instant when the block is at height H below the top of the sphere, while the block is still in contact with the sphere. Express your answer in terms of H, R, m, and g.

(**c**) [**2 points**] Show that the block leaves the spherical surface when it has dropped a vertical distance H of $R/3$.

Problem 3. [Total = 9 points] - Work done by a molecular motor in carrying a protein cargo inside a cell

Inside a cell is a molecular motor called "kinesin" (Figure 3A). It is a nano-sized machine made by a cell. It consists of several proteins. The kinesin looks like a miniature person. It has two feet that are connected together by miniature "legs". It also has a "tail", which are like your hands, that holds onto some protein. It walks forward on a "molecular road" called the "microtubule" inside cells. The kinesin moves by walking with its two feet, like a person. It does this while carrying the protein cargo with its tail. This is how proteins can be moved from one location to another inside a cell. There is no friction anywhere. There is also no gravity in this problem (kinesin and the protein cargo have tiny masses, so we can ignore gravity).

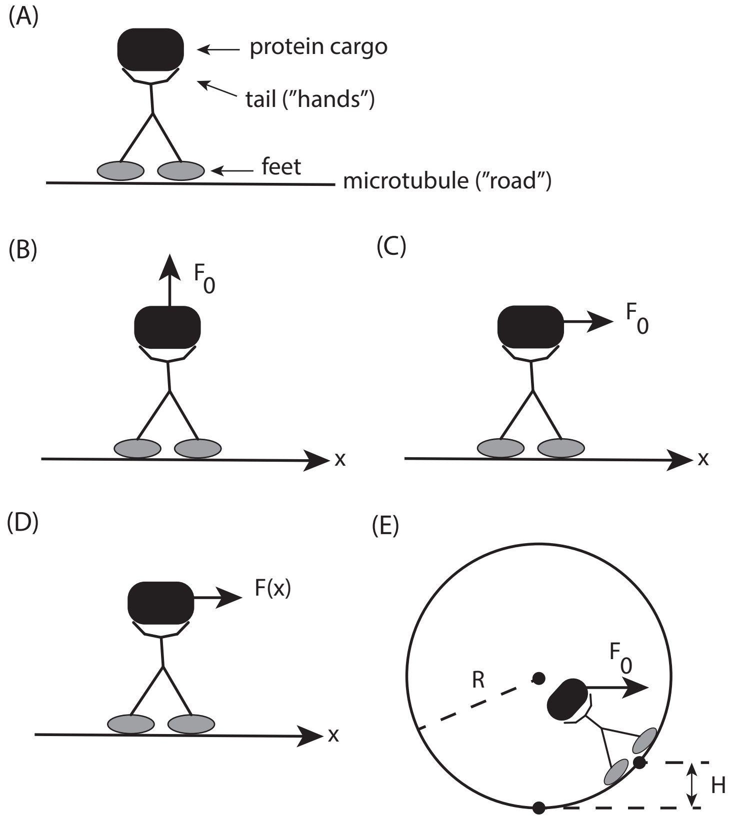

Figure 3: *Kinesin - a molecular motor inside a cell that carries a protein cargo and walks along a microtubule "road" (x-axis).*

(**a**) [**1 point**] Suppose that the kinesin exerts a constant upward force F_0 on the protein cargo that it carries (Figure 3B). It walks straight on the microtubule from position $x = 0$ to position $x = L$ along a straight microtubule road. The force it exerts is normal to the x-axis (i.e. normal to the microtubule road that it walks on). How much work is done by the kinesin on the protein cargo in this trip? Very briefly explain your reasoning.

(**b**) [**1 point**] Suppose that the kinesin exerts a constant force F_0 in the positive direction of the x-axis (Figure 3C). It walks straight on the microtubule from position $x = 0$ to position $x = L$. How much work is done by the kinesin in this trip?

(**c**) [**1 point**] Suppose that the kinesin exerts a force that *varies* as a function of its position on the microtubule (Figure 3D). It exerts a force

$$F(x) = ax \tag{1}$$

where $a > 0$. This force points towards the positive x-direction along the x-axis. What is the work done by the kinesin in moving from position $x = 0$ to position $x = L$?

(**d**) [**6 points**] Suppose that the kinesin exerts a constant force F_0 that always points towards the positive x-axis (Figure 3E). Now, the kinesin does not walk along a straight microtubule road along the x-axis. Instead, the microtubule forms a vertical circle (like in a rollercoaster). Assuming that there is no friction and no gravity, calculate the work done by the kinesin in carrying the protein cargo from the bottom of the circle to a final height H above the bottom of the circle ($H < R$) by walking up along the circle. Express your answer in terms of F_0, H, and R.

Problem 4. [Total = 11 points] - Gravitational energy and circular orbit

Consider two spherical planets, each of mass m, orbiting each other at a constant speed. The distance between them is R (Figure 4A).

(**a**) [**2 points**] Calculate the constant speed v of each planet's circular orbit in terms of R, m, and Newton's (universal) gravitational constant G (Figure 4A)

(**b**) [**2 points**] What is the total energy of this system of two orbiting planets? Express your answer in terms of R, m, and G (Figure 4A)

(**c**) [**3 points**] Can two planets of unequal masses orbit each other in a circle at the same constant speed, as in Figure 4A? If yes, calculate the period of the orbit. If not, explain your answer quantitatively.

(**d**) [**4 points**] Now consider three objects (Figure 4B). Two small planets, each of mass m, orbit circularly around the sun of mass M at a constant speed u. The sun is at the center of the circle. The distance of each planet from the

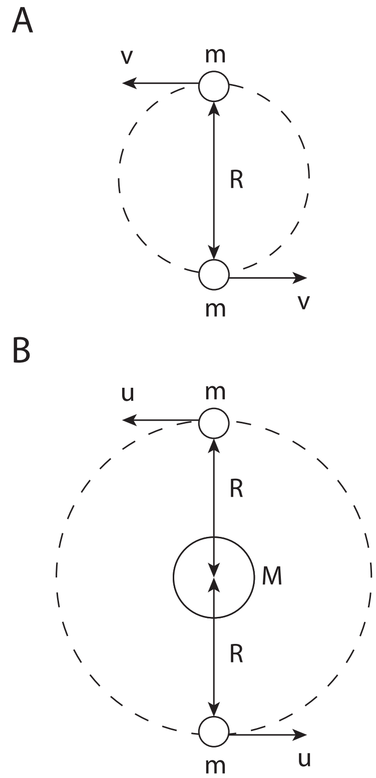

Figure 4: *Circular orbits*

sun is R. The sun does not move. Show that the period T of each planet's circular orbit is

$$T = 4\pi\sqrt{\frac{R^3}{G(4M+m)}} \tag{2}$$

Problem 5. [Total = 7 points] - Linear and rotational motion of a Nanocar

A recent advance in organic chemistry is the designing and making of *molecular machines*. Like the machines made of proteins inside every cell, nanobiologists and chemists have successfully made molecular machines inside test tubes with organic chemicals. For his part in making these pioneering nano-machines, Bernard Feringa at the University of Groningen has been awarded this year's Nobel prize in Chemistry. Here we consider a simplified view of the motion of the nanocar that his group has designed (Figure 5).

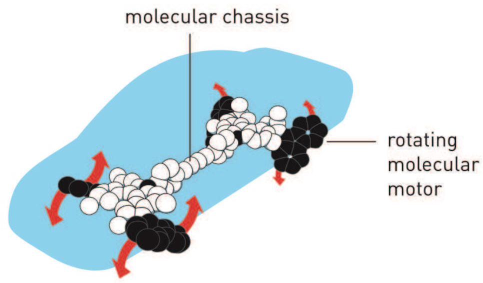

Figure 5: *Nanocar - Designed by Bernard Feringa's group at Groningen (Image from this year's Nobel press release)*

The nanocar works as follows: You shine a UV light. The light causes the car's wheels to rotate. The car's wheels reach a constant angular speed ω (in radians / time) immediately after the light is turned on. This causes the car to move forward at a **constant speed** V towards the positive x-axis immediately after the light is turned on. When you turn off the UV light, the car immediately stops moving. The car can only move while the UV light is shining on it. We assume here that the car moves only in one dimension, along the x-axis.

Suppose the car is initially at position $x = 0$. It is initially at rest. You then turn on a UV light and shine it on the car for a time duration of ΔT. Then you shut off the light for a time duration ΔT. Then you turn on the light again for a time duration ΔT, you turn off the light. And this keeps going on and on.

(**a**) [**2 points**] For the motion that takes place for a time duration of $2N\Delta T$, what is the nanocar's average velocity? Here N is a positive integer ($N \geq 1$).
[*Hint* - Remember that $V_{average}\times$ (total travel time) = total distance travelled]

(**b**) [**1 point**] Suppose that each wheel has a radius R. Assume that the nanocar moves without the wheels slipping on the road (so if the wheel rotates once, the car moves forward by a distance $2\pi R$). For the motion that takes place for a time duration of $2N\Delta T$, What is the average angular speed of the wheels?

(c) [**2 points**] How many times does the wheel rotate during a time duration of $2N\Delta T$.

(d) [**1 point**] When a UV light shines on the car, how much work is done by the light to drive the car? Assume that the nanocar has mass m and that all the energy from light is transferred to the car immediately after the light is turned on.

(e) [**1 point**] Over the time duration of $2N\Delta T$, how much energy is lost in total as heat?

Problem 6. [Total = 8 points] - Toy train falling off a table

Consider a toy train that consists of three blocks of masses m_1, m_2, and m_3. Block of mass m_1 is joined to a block of mass m_2 by a massless rope. The block of mass m_2 is joined to the block of mass m_3 by a massless Hookian spring that has a spring constant k. The force of gravity is constant in this problem. The constant acceleration due to gravity is $\vec{g}$ (a vector that points downwards). Let $g = |\vec{g}|$ (a positive number that is the magnitude of the acceleration due to gravity). The rope that connects m_1 with m_2 is wrapped around a massless and frictionless pulley wheel.

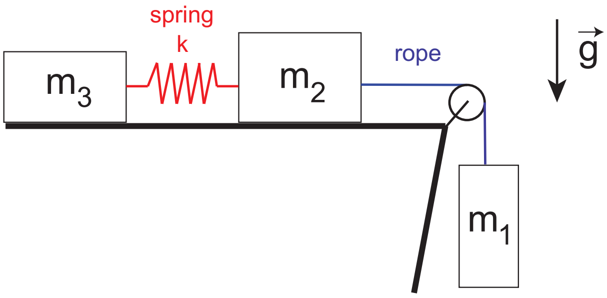

Figure 6: *Three blocks with a massless and frictionless pulley*

Initially, you hold everything with your hands so every block is at rest. Then you release all three blocks. After some time, all three blocks move together with the **same magnitude of acceleration**. There is no friction anywhere in this problem. Assume that you observe the block of mass m_1 moving **downwards**.

(a) [**2 points**] For each block, draw a diagram that shows all the forces acting on that block. No need to calculate the forces here. Just indicate the type of force for each force arrow (e.g. gravity, spring, tension).

(b) [**2 points**] What is the magnitude of the acceleration of each block? Assume that m_1 is accelerating downwards.

(c) [**2 points**] Calculate the tension T in the rope that joins m_1 to m_2.

(d) [**2 points**] Calculate the length by which the spring is compressed or stretched relative to its rest length. Also indicate if the spring stretched or compressed.

(e) [**Bonus - 2 points (only if you get this completely correct)**] Calculate the acceleration *vector* of the center of mass while the masses m_2 and m_3 remain on the table during their acceleration (i.e. before the masses m_2 and m_3 fall off the table).

(f) [**Bonus - 2 points (only if you get this completely correct)**] What is the force that the pulley exerts on the system? Express the force as a vector (it has two components).