NB1140: Physics 1A - Classical mechanics and Thermodynamics
Midterm exam - Potentially useful formulae
Wednesday 14 December 2016
9:00 - 12:00 (3 hours)

Center of mass' position:

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_N\vec{r}_N}{m_1 + m_2 + ... + m_N} \tag{1}$$

where m_i and $\vec{r}_i$ are the mass and the position vector of the i-th particle respectively. $\vec{r}_{cm}$ is the position vector of the system's center of mass.

Linear momentum $\vec{p}$:

$$\vec{p} = m\vec{v} \tag{2}$$

where m and $\vec{v}$ are the mass and velocity of the particle respectively.

Energies:

Gravitational potential energy of a particle of mass m, at a distance of r away from a particle of M is

$$E_{grav} = \frac{-GMm}{r} \tag{3}$$

where we have set the zero of the potential energy (i.e. the "reference point") to be at $r = \infty$ and G is the universal (Newton's) gravitational constant. But if the particle is close to the surface of the Earth, at a height h above the Earth (or any surface), is

$$E_{grav} = mgh \tag{4}$$

where g is the constant acceleration due to gravity (g = 9.8 m/s^2).

The kinetic energy of a particle of mass m that travels at speed v is

$$KE = \frac{mv^2}{2} \tag{5}$$

The potential energy of a Hookian spring that has a spring constant k and has a compressed or stretched length of x compared to its rest length is

$$PE_{spring} = \frac{kx^2}{2} \tag{6}$$

If you apply a force $\vec{F}$ to an object while the object moves an infinitesimal displacement $\vec{dx}$, then the work dW that you do on the object is the dot product between the two vectors:

$$dW = \vec{F} \cdot \vec{dx} \tag{7}$$

If the object moves a finite distance (i.e. more than just an infinitesimal distance), say from an initial position r_0 to a final position r_f, then the total work W done by you on the object in this trip is:

$$W = \int_{r_0}^{r_f} \vec{F}(\vec{r}) \cdot \vec{dr} \tag{8}$$

where $\vec{F}(\vec{r})$ is the force that you exert on the object when the object is at position $\vec{r}$ and the integral is along the travel path taken by the object from position $\vec{r}_0$ and ends at position $\vec{r}_f$.

Forces:

The Hookian spring with a spring constant k ($k > 0$) that is stretched or compressed from its rest length by *displacement* x exerts a force F_{spring}:

$$F_{spring} = -kx \tag{9}$$

Note that x can be positive or negative.

The Newton's law of gravity says that the force that a particle of mass m_1 exerts on a particle of mass m_2 that is a distance r away from it is

$$F_{grav} = -\frac{Gm_1 m_2}{r^2}\hat{r} \tag{10}$$

where $\hat{r}$ is the unit vector that starts from the position of particle of mass m_1 and points towards the location of the particle of mass m_2.

A particle of mass m that is moving in a circle of radius r at a uniform speed v experiences a centripetal force $\vec{F}_{cent}$ whose magnitude is

$$|\vec{F}_{cent}| = \frac{mv^2}{r} \tag{11}$$

If the coefficient of kinetic friction is μ_k, then on a block that is moving, the frictional force on it is

$$F = \mu_k F_N \tag{12}$$

where F_N is the normal force that the surface exerts on the block in a direction that is perpendicular to the surface, away from the surface.

Kinematics:

Rotational speed ω of a circular wheel of radius R is related to the wheel's linear speed v by

$$R\omega = v \tag{13}$$

Velocity $\vec{v}$ of a particle is defined as

$$\vec{v} = \frac{d\vec{r}}{dt} \tag{14}$$

where $\vec{r}$ is the position vector. The acceleration $\vec{a}$ of a particle is defined as

$$\vec{a} = \frac{d\vec{v}}{dt} \tag{15}$$